ANNALS of Faculty Engineering Hunedoara – International Journal of Engineering
Tome XX [2022] | Fascicule 3 [August]

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STABILITY STUDIES OF STRAIGHT BEAMS USING FINITE ELEMENT METHOD

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Abstract: In this paper, the buckling analysis of pinned–pinned beams with an intermediate roller support and pinned–pinned beams with an intermediate spring support is carried out using the finite element method. The results for the critical loads for one–, two– and three–dimensional models with homogeneous, as well as heterogeneous cross–sections are investigated and compared. In order to illustrate the validity and accuracy of the presented method, the critical force findings are compared with those found in the literature. A good agreement is concluded in each case.
Keywords: FEM; buckling; beam; axial load

1. INTRODUCTION

The finite element method (FEM) is a powerful tool that offers a vast number of applications – see, e.g., [3,4,6]. Many studies have been developed by researchers regarding buckling since Euler's pioneering work. Book [7] offers a thorough examination of the theory of elastic stability of continuously axially loaded columns. Moreover, the static and dynamic stability of columns under self–weight is addressed in [9] by means of analytical and numerical studies. Anisotropic laminated columns are in the spotlight in [1]. The authors present analytical and FEM parametric studies too. In [8], the buckling instability of a system of three simply supported elastic Timoshenko beams linked by Winkler elastic layers with each beam subjected to the same compressive axial force was examined. Using the Ritz method, in paper [2] Aydogdu investigated the buckling of cross–ply laminated beams under various boundary circumstances. The authors of [5] presented the issue of how the critical load of heterogeneous beams with three supports can be determined provided that the intermediate support is a spring. They used Green's function of the three–point boundary value problem to solve the eigenvalue problem that provides the critical loads using the boundary element method (BEM).

In the present paper, a finite element method based approach is applied to study the buckling behavior of pinned–pinned beams with an intermediate roller support (PrsP) and with an intermediate spring support (PssP) subject to axial load. The material is linearly elastic, isotropic and the material distribution can change over the cross–section – it is called cross–sectional inhomogeneity. As it turns out, the buckling loads are affected significantly by the location of the middle support. The results of the critical loads are compared to those found using the boundary element technique and there is a high correlation observed.

2. ADDRESSING THE STABLITY PROBLEMS

The considered beams are shown in Figure 1. The axial external force N is a compressive load applied on the right end side of each beam. The beams have uniform cross–section throughout their length. The E–weighted centerline of the beams coincide with the axis $\hat{x}$. It is assumed that the coordinate plane $\hat{x}\hat{z}$ is a symmetry plane of the beams. When it is about heterogeneous cross–section, the modulus of elasticity E satisfies the symmetry condition E $(\hat{y}, \hat{z})$= E $(-\hat{y}, \hat{z})$. The beam's initial length is L, and the location of the middle support is identified by $\hat{b}$, which value is between 0 and 1. When zero, the left and intermediate supports coincide, when 1, the right and intermediate support positions are the same.

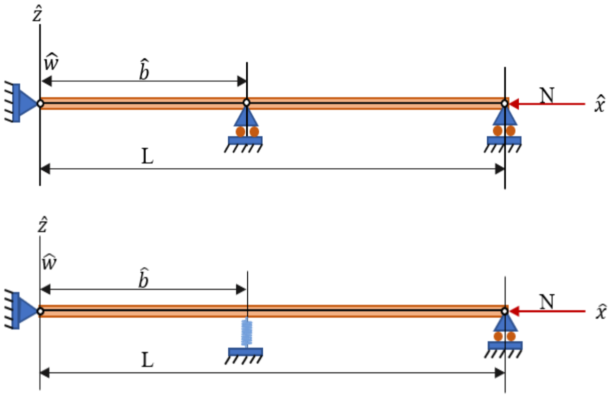

Figure 1. (a) PrsP beam; (b) PssP beam

Consider now the bi–material beam with cross–section shown in Figure 2. It is assumed that a=c=10 mm, $a_1 = a_2 = a/3$, L=100 mm, $E_1=E_{Steel}$=2e5 N/mm^2 , $E_2=E_{Aluminum}$ =0.7e5 N/mm^2 and, in accordance with the finite element software documentation, to find the buckling load, a total of N= 1 N unit force is applied on the right end of each beam. In the 1D model, it is a concentrated point load, in 2D and 3D, it is uniformly distributed over the end–section.

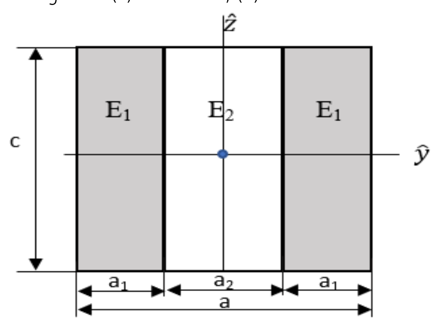

Figure 2. Cross-section of the selected heterogeneous beams

When the beams are heterogeneous, the flexural stiffness to axis $\hat{y}$ is

$$I_{ey} = \frac{ac^4}{12}\left[\frac{2E_1+E_2}{3}\right] = \frac{10^4}{12}\left[\frac{2.2+0.7}{3}\right] * 10^5 = 1.3056 * 10^8 \text{ Nmm}^2 . \quad (1)$$

However, when the beams are made of homogenous steel, the flexural stiffness becomes

$$I_{ey} = IE_1 = \frac{10^4}{12} * 2 * 10^5 = 1.6666 * 10^8 \text{ Nmm}^2 . \quad (2)$$

(For pure aluminium, it is 0.58e6 Nmm^2.) In the 1D model, the average Young modulus and Poisson number meant the input data for cross–sectional heterogeneity, thus

$$E = \left[\frac{2*E_1+E_2}{3}\right] = \left[\frac{2.2+0.7}{3}\right] * 10^5 = 1.5667 * 10^5 \text{ N/mm}^2, \quad (3)$$

$$\nu = \left[\frac{2\nu_1+\nu_2}{3}\right] = \left[\frac{2*0.3+0.33}{3}\right] = 0.31. \quad (4)$$

Commercial software Ansys was used to calculate the critical loads of the considered beams. The adaptive finite element approach was used to identify sections of the mesh where the solution is insufficiently precise. In these ranges, the mesh was refined until the solution achieved the required degree of precision. The published results can be considered as converged ones. Because of the applied kinematic restraints, buckling can only occur about axis $\hat{y}$. The layers are perfectly tied.

— Stability problems of PrsP beams

In this Section, the numerical results achieved about pinned–pinned beams with an intermediate roller support are presented. Both homogeneous, and heterogeneous findings are given, as per the 1D/2D/3D models – see Tables 1–3. The first buckling modes are illustrated in Figures 3–5.

Table 1. Critical load results of 1D PrsP beams

1D homogeneous PrsP beam			1D heterogeneous PrsP beam		
$\hat{b}(-)$	FEM (10^5 N)	BEM (10^5 N) [5]	$\hat{b}(-)$	FEM (10^5 N)	BEM (10^5 N) [5]
0	1.604	1.643	0	1.256	1.287
0.25	4.456	4.878	0.25	3.560	3.821
0.5	5.978	6.573	0.5	4.775	5.149
0.75	4.456	4.878	0.75	3.560	3.821
1	1.604	1.643	1	1.256	1.287

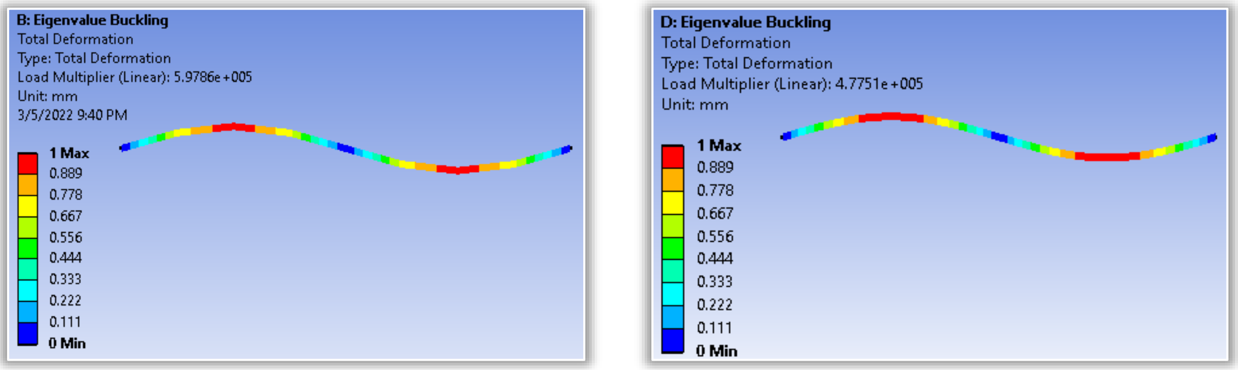

Figure 3. First buckling mode shape when $\hat{b} = 0.5$:(a) 1D PrsP with homogeneous cross–section;(b) 1D PrsP with heterogeneous cross section

In the present problem, buckling can only occur about the $\hat{y}$ axis when there are three vertical layers and because of the constraints. Hence the 2D heterogeneous scenario can't be modelled as a plane stress state since the material distribution is a function of $\hat{y}$.

Table 2. Critical load results of 2D PrsP beams

2D homogeneous PrsP beam		
$\hat{b}(-)$	FEM (10^5 N)	BEM (10^5 N) [5]
0	1.646	1.643
0.25	4,833	4.878
0.5	6,604	6.573
0.75	4,833	4.878
1	1.646	1.643

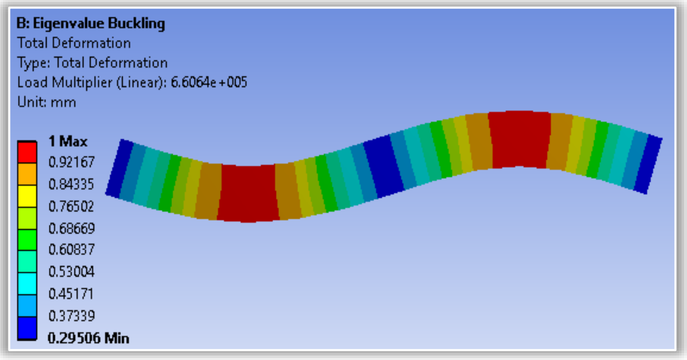

Figure 4. First buckling mode shape of 2D homogeneous PrsP beam ($\hat{b} = 0.5$)

Table 3. Critical load results of 3D PrsP beams

3D homogeneous PrsP beam			3D heterogeneous PrsP beam		
$\hat{b}(-)$	FEM (10^5 N)	BEM (10^5 N) [5]	$\hat{b}(-)$	FEM (10^5 N)	BEM (10^5 N) [5]
0	1.632	1.643	0	1.281	1.287
0.25	4.767	4.878	0.25	3.742	3.821
0.5	6.391	6.573	0.5	5.013	5.149
0.75	4.767	4.878	0.75	3.742	3.821
1	1.632	1.643	1	1.281	1.287

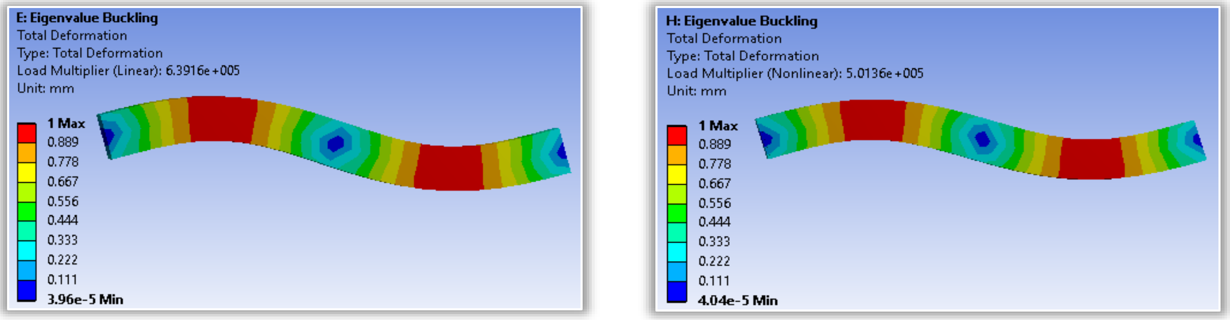

Figure 5. First buckling mode shape when $\hat{b} = 0.5$:(a) 3D PrsP with homogeneous cross–section; (b) 3D PrsP with heterogeneous cross–section

— Stability problems of PssP beams

The numerical results for the second case are gathered hereinafter.

Table 4. Critical load results of 1D PssP

1D homogeneous PssP beam			1D heterogeneous PssP beam		
$\hat{b}(-)$	FEM (10^5 N)	BEM (10^5 N) [5]	$\hat{b}(-)$	FEM (10^5 N)	BEM (10^5 N) [5]
0	1.604	1.643	0	1.256	1.287
0.25	3.526	3.710	0.25	2.958	3.087
0.5	5.973	6.573	0.5	4.775	5.149
0.75	3.526	3.710	0.75	2.958	3.087
1	1.604	1.643	1	1.256	1.287

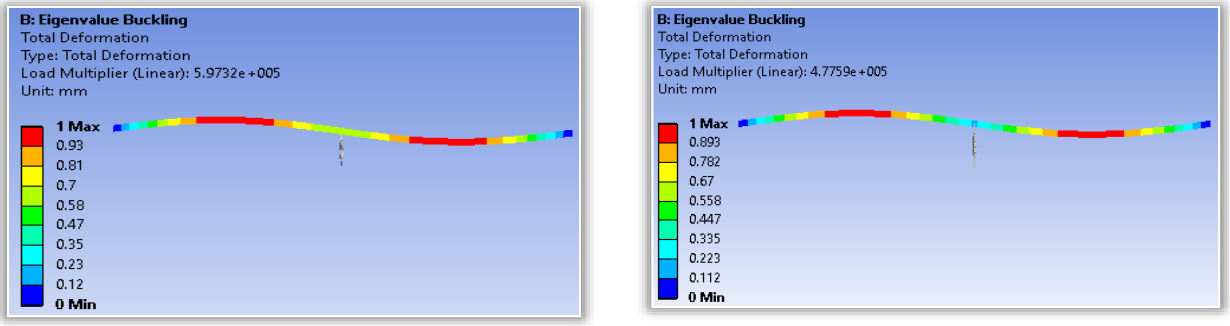

Figure 6. First buckling mode shape when $\hat{b} = 0.5$:(a) 1D PssP with homogeneous cross–section;(b) 1D PssP with heterogeneous cross section

Table 5. Critical load results of 2D PssP

2D homogeneous PssP beam		
$\hat{b}(-)$	FEM (10^5 N)	BEM (10^5 N) [5]
0	1,646	1.643
0.25	3,650	3.710
0.5	6,427	6.573
0.75	3,650	3.710
1	1,646	1.643

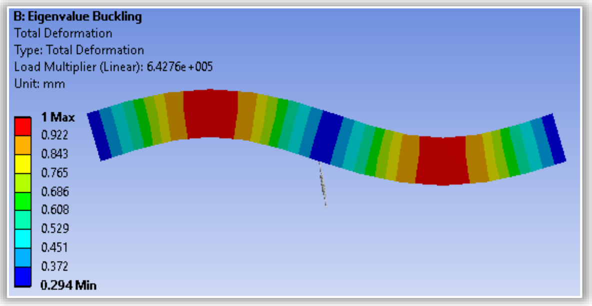

Figure 7. First buckling mode shape of 2D PssP with homogeneous cross–section ($\hat{b} = 0.5$)

To study the stability of PssP beams shown in Figure 1b, it was assumed that the stiffness of the spring is k=33333 N/mm. For all the simulations, the spring support has two degrees of freedom (translation along $\hat{z}$ axis and rotation about $\hat{y}$ axis. Similarly, as before, the buckling loads are evaluated according to multiple cross–sections and models. Again, as 2D PrsP beams, the heterogeneous case can't be modelled properly.

Table 6. Critical load results of 3D PssP

3D homogeneous PssP beam			3D heterogeneous PssP beam		
$\hat{b}(-)$	FEM (10^5 N)	BEM (10^5 N) [5]	$\hat{b}(-)$	FEM (10^5 N)	BEM (10^5 N) [5]
0	1.632	1.643	0	1.281	1.287
0.25	3.791	3.710	0.25	3.233	3.087
0.5	6.728	6.573	0.5	5.327	5.149
0.75	3.791	3.710	0.75	3.233	3.087
1	1.632	1.643	1	1.281	1.287

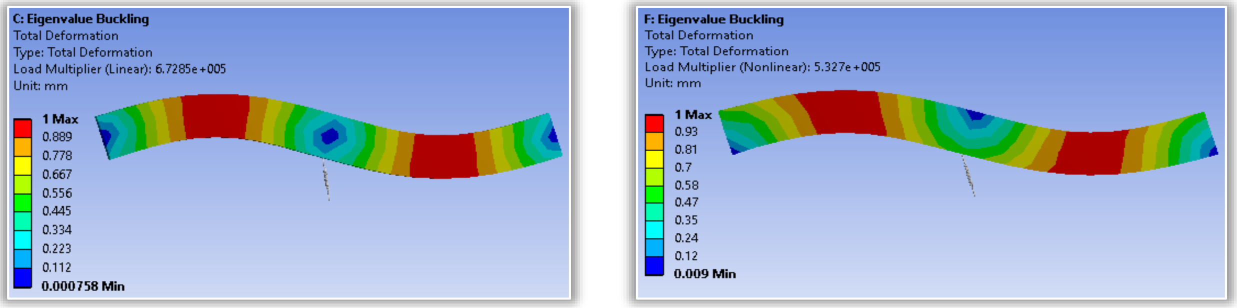

Figure 8. First buckling mode shape when $\hat{b} = 0.5$:(a) 3D PssP with homogeneous cross–section;(b) 3D PssP with heterogeneous cross–section.

Table 7 shows the relationship between the critical loads and the spring support stiffness when b=0.25. If the spring stiffness (k) tends to zero, the PssP beam acts as if it were a pinned–pinned beam and if k tends to infinity, the PssP beam behaves as if it were a pinned–pinned one beam with an intermediate rigid roller support. These results and remarks were clarified in details by the authors in the paper [4] using the boundary element method with the help of the Green function technique.

Table 7. Variation of the buckling load of 3D PssP beams against k when $\hat{b} = 0.25$

Spring stiffness k (N/mm)	FEM	
	3D homogeneous PssP beam	3D heterogeneous PssP beam
10	1.788	1.469
100	1.796	1.478
1000	1.877	1.558
10000	2.601	2.253
33333	3.791	3.233
50000	4.224	3.528
100000	4.758	3.861
500000	5.306	4.126
5000000	5.311	4.183

3. CONCLUSIONS

A finite element method was used to study the stability problem of PrsP and PssP beams. The most important conclusions are gathered below.

- 3D and 2D elements give better results of the critical loads than that the 1D elements because they give the best insight to the deformation state of the structure.
- For all the simulation cases, the maximum critical load has been found when the roller/spring support is located in the middle.
- The spring stiffness has also an effect on the critical load. When the spring stiffness tends to infinity, the PssP beam reacts like PrsP beam. When the spring stiffness is zero, it is the case when only the two end supports are present.
- Material heterogeneity also has notable effects on the critical loads. Therefore with suitable material selection, it is possible to increase the critical loads.
- The results of the critical loads by means of FEM were compared to those found using the boundary element technique. The agreement is judged to be really good.

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