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CALCULATION OF OPTIMAL STRUCTURES MADE OF ARTICULATED BARS

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Abstract: The optimization is stimulated by the need to improve the conditions of use and operation of the structures, simultaneously with the requirements to reduce the size and weight of the structures. In the paper is presented in the calculation of articulated structures the criterion of equal strength, which determines the structure that meets the minimum volume condition. In the second paragraph of the paper starting from the volume formula of a statically indeterminate articulated structure depending on the tension in the bar, the bar length and the total number of bars through a series of mathematical calculations we arrive at the volume formula for a statically determined articulated structure. Next, the minimum volume expression is calculated taking into account some mathematical restrictions and conditions. The procedure is continued until the minimum volume values, calculated for two successive steps, coincide. It is still considered an articulated structure once statically indeterminate given by an equation. This equation is represented graphically in the xOy plane by a polygonal line and each point of intersection of the segments that make up the polygonal line is a solution of some previous equations. Some auxiliary functions are introduced in order to solve the volume equation. An example is a metal articulated structure that has the efforts given in a table. Depending on the volume of the articulated structure and its minimum, the efforts and areas of the sections of the minimum volume bars are obtained. In conclusion, the calculation of the optimal structures made up of articulated bars, from the condition of minimum volume, determines a structure of minimum volume which is at the same time a structure of equal resistance. Some data in the tables show the difference between the volume of the structure subjected to stresses, if the stresses in its bars were calculated using the stress method and if they were calculated with the minimum volume condition. At the same time, the compatibility condition is respected, the equations that express this condition being verified.
Keywords: articulated structure, optimization, criterion of equal strength, minimum volume condition

1. INTRODUCTION

Lattice beams are systems of rigid bars hinged at the ends. These joints are called nodes. If the truss bars are contained in same plane, the beams are called plane, otherwise they are called spatial. In construction, lattice beams are structural elements, which have the role of a taking loads from other structural or structural elements and to them transmit to the supports. They are used on roofs with large openings, bridges, industrial buildings etc. In mechanics, the lattice beam is two physical models: systems of material points, systems of bodies. To be functional, lattice beams must meet two conditions: to be non-deformable from a geometric point of view and to be fixed on the support bodies. In the paper is presented in the calculation of articulated structures the criterion of equal strength, which determines the structure that meets the minimum volume condition. Starting from the volume formula of a statically indeterminate hinged structure depending on the tension in the bar, the length of the bar and the total number of bars through a series of mathematical calculations we arrive at the volume formula for a determined hinged bar. Some data in the tables show the difference between the volume of the structure subjected to stresses, if the stresses in the bars were calculated by the tension method and if they were calculated with the minimum volume condition. At the same time, the compatibility condition is respected, checking the equations that express this condition.

2. THEORETICAL ASPECTS AND RESULTS

The volume of a statically indeterminate articulated structure is given by the expression:

$$V = \sum_{k=1}^{m} \frac{|N_k| \cdot l_k}{\sigma_k} \quad (1)$$

where N_k is the tension in bar k, l_k is the length of bar k, σ_k is the admissible stress in bar k, and m is the number of bars.[2]

Having:

$$N_k = N_k^0 + \sum_1^n n_{ki} \cdot X_I$$

where N^0_k represents the tension in bar k in the base system, required under external loading n_{ki} (i = 1, …, n) the stresses in the bars of the basic system due to the unit unknowns $X_i = 1$ (I = 1, …, n), the volume expression becomes:

$$V = \sum_{k=1}^{m} \left| a_k + \sum_{i=1}^{n} b_{ki} \cdot X_i \right| \quad (2)$$

in which:

$$a_k = \frac{N_k^0 \cdot l_k}{\sigma_k}, \qquad b_{ki} = \frac{n_{ki} \cdot l_k}{\sigma_k}.$$

For a statically determined articulated structure, one obtains:

$$V = \sum_{k=1}^{m} a_k = V_0$$

Since this base system was arbitrarily chosen, V_0 is not necessarily minimal.[2]

Considering $X_1 \neq 0$, $X_2 = X_3 = \cdots = X_n = 0$, it is obtained:

$$V = \sum_{k=1}^{m} |a_k + b_{kl} \cdot X_1| \qquad (3)$$

The minimum of this expression is determined as follows:

≡ the roots of each expression under the "absolute value" sign are obtained by successive cancellation of all terms

$$a_i + b_{i1} \cdot X_1 = 0 \qquad (i = 1, \dots, m) \qquad (4)$$

≡ equations (4) are performed so that the roots are in ascending order;

≡ the expression is calculated

$$\frac{1}{2}\sum_{k=1}^{m} b_{k1}$$

and the coefficient b_{i1} is determined, for which the inequality:

$$|b_{11}| + |b_{21}| + \cdots + |b_{i1}| \geq \frac{1}{2}\sum_{1}^{m} |b_{k1}| \qquad (5)$$

to be verified.

In this case, the value of the unknown X_1, determined from ecuation $a_l+b_{l1}=0$ minimize expression (3)

Denoting this value by $X_1^{(1)}$, the corresponding volume

$$V = \sum_{k=1}^{m} |a_k + b_{k1} \cdot X_1^{(0)}| = V_1$$

is minimal.

To show that $V_1 \geq V_0$, calculate:

$$\sum_{k=1}^{m} |a_k + b_{k1} \cdot X_1^{(1)}| = \sum_{k=1}^{m} \left|a_k - \frac{a_l \cdot b_{kl}}{b_{l1}}\right| = \sum_{k=1}^{l} \left|a_k - \frac{a_l \cdot b_{kl}}{b_{l1}}\right| + \sum_{k=l+1}^{m} \left|a_k - \frac{a_l \cdot b_{kl}}{b_{l1}}\right|$$

The roots obtained by solving equations (4) lead to the following inequalities:

$$-\frac{a_1}{b_{1l}} < -\frac{a_2}{b_{21}} < \cdots < \frac{a_l}{b_{l1}} < -\frac{a_{l+1}}{b_{l+1,1}} < \cdots < -\frac{a_m}{b_m}$$

otherwise

$$\frac{a_1}{b_{11}} > \frac{a_2}{b_{21}} > \cdots > \frac{a_l}{b_{l1}} > \cdots > \frac{a_m}{b_{ml}}$$

For $l \leq k \leq m$ is obtained $\frac{a_l}{b_{l1}} > \frac{a_k}{b_{k1}}$

From here the inequality is deduced:

$$V_1 \leq \sum_{k=1}^{l} |a_k| + \sum_{k=l+1}^{m} \left|\frac{a_l \cdot b_{k1}}{b_{l1}}\right| = \sum_{k=1}^{l} |a_k| + \left|\frac{a_l}{b_{l1}}\right| \sum_{k=l+1}^{m} |b_{k1}|$$

Condition (5) leads to:

$$\sum_{k=1}^{l} |b_{k1}| \geq \frac{1}{2}\sum_{k=1}^{l} |b_{k1}| + \frac{1}{2}\sum_{k=l+1}^{m} |b_{k1}|$$

otherwise

$$\sum_{k=1}^{l} |b_{k1}| \geq \sum_{k=l+1}^{m} |b_{k1}|$$

So, it was shown that:

$$V_1 \leq \sum_{k=1}^{m} |a_k| = V_0$$

Considering $X_1 = X_1^{(1)}, \; X_2 \neq 0, \; X_3 = X_4 = \cdots = X_n = 0$, it is obtained:

$$V = \sum_{k=1}^{m} |a_k + b_{k1} \cdot X_1^{(1)} + b_{k2} \cdot X_2|$$

Calculating the minimum of this expression as before, a value $X_2^{(2)}$ is determined for which the volume V is minimum.

If this value is denoted by V_2 we will have:

$$V_2 \leq V_1 \leq V_0$$

Continuing the procedure, a string of $V_0, V_1, \ldots, V_s$ values is determined for which:

$$V_s \leq V_{s-1} \leq \cdots \leq V_2 \leq V_1 \leq V_0.$$

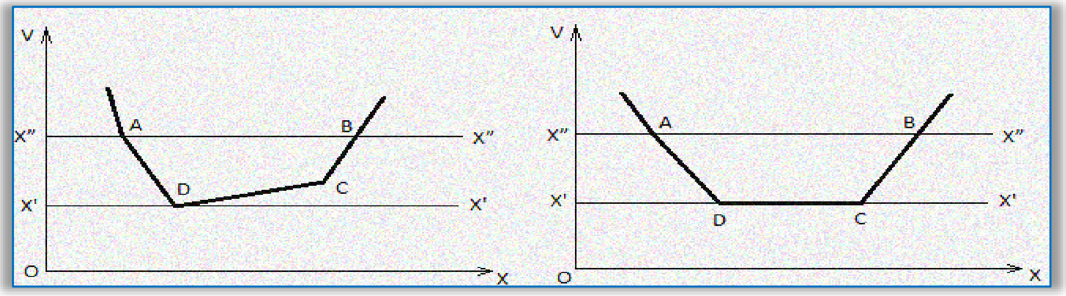
The process continues until the minimum volume values, calculated for two successive steps, coincide.

It was considered an articulated structure once statically indeterminate. The volume is given by the equation:

$$V = \sum_{k=1}^{m} |a_k + b_{k1} \cdot X|$$

This equation is represented graphically in the (X,Y) plane by the polygonal line in figure.1.a. Each point of intersection of the segments that make up the polygonal line represents a solution of equations (4).

If the limit value of the minimum volume is reached in point D (figure.1.a) or on segmental DC (figure.1.b), then through point D or segmental DC there is a tangent X'X', parallel to the OX axis. Otherwise, the X"X" line intersects the polygon line at two points A and B.



a) b)

Figure 1. Representation of the minimum volume

The problem is reduced to solving equation (6) which can be put in the form:

$$V = \sum_{k=1}^{m} |b_{k1}||X - A_k|, \qquad \frac{a_k}{b_{k1}} = A_k \tag{6}$$

Entering the function:

$$\Phi(A_k) = \frac{1}{2}\left(1 + \frac{|X - A_k|}{X - A_k}\right), \tag{7}$$

which takes the values:

$$\Phi(A_k) = \begin{cases} 0 & X \le A_k \\ 1 & X > A_k \end{cases},$$

from (7) is determined:

$$|X - A_k| = (2\Phi(A_k) - 1) \cdot (X - A_k)$$

In this case, the volume equation becomes:

$$V = \sum_{k=1}^{m} |b_{k1}|(2\Phi(A_k) - 1) \cdot (X - A_k)$$

Solving this equation with respect to X gives:

$$X = \frac{\sum_{k=1}^{m} 2|b_{k1}| \cdot A_k \Phi(A_k) - \sum_{k=1}^{m} |b_{k1}| \cdot A_k + V}{\sum_{k=1}^{m} 2|b_{k1}| \Phi(A_k) - \sum_{k=1}^{m} |b_{k1}|} \tag{8}$$

Taking $X \le A_k$ or $X > A_k$, for any value of k, the unknown X is determined from equation (8). The calculation process continues until all Ak values are exhausted.[2]

In solving equation (8), the following possible situations appear:

- if the equation has only one solution for any k, the value of V is the limiting value of the minimum volume;
- if the equation admits two sufficiently close roots, an infinity of solutions is obtained;
- if the equation admits two roots that do not have the same order of magnitude, the problem has no solutions.

It is now shown that the hinged structure of minimum volume is of equal strength. The volume of the structure is given by:

$$V = \sum_{k=1}^{m} A_k \cdot l_k$$

in which:

$$A_k = \frac{|S_k|}{If_k}, \tag{9}$$

represents the area of the base.

The minimum volume condition (9) determines a minimum volume structure that is also of equal strength.

The system of compatibility equations is:

$$\sum_{j=1}^{n} u_{ij} \cdot X_j + u_i^0 = 0 \quad (i = 1, \dots, n), \quad u_{ij} = \sum_{k=1}^{m} \frac{n_{ki} \cdot n_{kj}}{E \cdot A_k}, \quad u_i^0 = \sum_{k=1}^{m} \frac{N_k^0 \cdot n_{ki}}{E \cdot A_k} \tag{10}$$

Substituting (9) into the expressions (10) we obtain, for σ=constant:

$$u_{ij} = \sigma \cdot \sum_{k=1}^{m} \frac{\mathrm{sign} M_k \cdot n_{kj} \cdot n_{ki}}{E \cdot S_k}, \qquad u_i^0 = \sigma \cdot \sum_{k=1}^{m} \frac{\mathrm{sign} M_k \cdot N_k^0 \cdot n_{ki}}{E \cdot S_k}$$

and, equations (10) become:

$$\sum_{k=1}^{m} \mathrm{sign} M_k \cdot n_{ki} = 0, \quad (i = 1, \dots, m)$$

which are identical to the minimum volume equations

$$\frac{\partial V}{\partial X_i} = 0, \quad (i = 1, \dots, n)$$

As an example, consider the metal articulated structure in figure 2. In table 1 the efforts N^0 and n are given. The volume of the articulated structure is given by the expression (2).

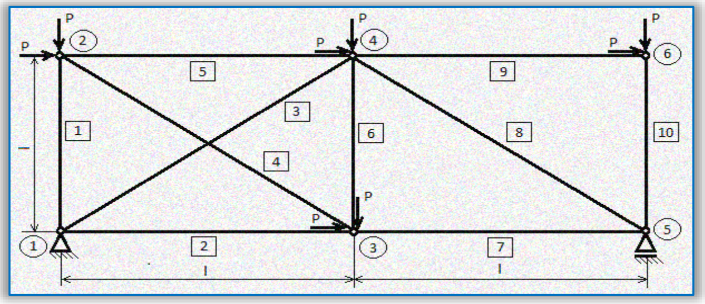


Figure 2. Metal articulated structure

Table 1. The efforts N^0 and n

Bar	N^0	n	N	Minimum volume	
				N	A[cm²]
1-2	-P	$-\frac{\sqrt{2}}{2}$	-3,72P	-P	2
1-3	$\frac{7P}{2}$	$-\frac{\sqrt{2}}{2}$	3,28P	4P	8
1-4	$\frac{P\sqrt{2}}{2}$	1	1,03P	0	0
2-3	0	1	0,32P	-0,707P	1,414
2-4	-P	$-\frac{\sqrt{2}}{2}$	-1,23P	-P	2
3-4	P	$-\frac{\sqrt{2}}{2}$	0,78P	1,5P	3
3-5	$\frac{5P}{2}$	0	2,5P	2,5P	5
4-5	$\frac{-5\sqrt{2}P}{2}$	0	-3,53P	-3,53P	7,06
4-6	P	0	P	P	2
5-6	-P	0	-P	-P	2

$$V = \frac{1}{lf}\left(\left|-P - \frac{\sqrt{2}}{2}X_1\right| + 3\left|\frac{7P}{2} - \frac{\sqrt{2}}{2}X_1\right| + 3\sqrt{2}\left|\frac{P\sqrt{2}}{2} + X_1\right| + 3\sqrt{2}|X_1| + 3\left|-P - \frac{\sqrt{2}}{2}X_1\right| + 3\left|P - \frac{\sqrt{2}}{2}X_1\right| + 3\frac{5P}{2} + 3\frac{5\sqrt{2}}{2}P + 3P + 3P\right)$$

The minimum of this expression is reached for the value: $X_1 = -\frac{P\sqrt{2}}{2}$

For P = 30kN, σ = 1500daN/cm² and 1 = 3 m, the forces and areas of the minimum volume bar sections given in table 2 are obtained. The minimum volume has the value $0.112 \cdot 10^6$ cm³.

3. CONCLUSIONS

The calculation of the optimal structures made of articulated bars, from the condition of minimum volume, determines a structure of minimum volume which is at the same time a structure of equal strength. Table 1 shows the difference between the volume of the structure subjected to stress, if the efforts in its bars were calculated with the effort method and if they were calculated with the minimum volume condition. At the same time, the compatibility condition is respected, the equations expressing this condition being verified.

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