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[1.]Corneliu BOŢAN, [1.]Vasile HORGA

# OPTIMAL CONTROL OF THE DRIVE SYSTEMS WITH SPEED DEPENDENT LOAD TORQUE

[1.] "Gheorghe Asachi" Technical University of Iaşi, ROMANIA

**Abstract:** The paper deals with the optimal control of the electrical drive systems with linear dependent on speed load torque. The minimum of the Joule losses in the transient period is imposed. Different types of motors are considered. The approach refers to the current control. Two types of problems depending on the final conditions are considered. The optimization is done in the sense of minimization of the losses in the transient period. Taking into account the great values of the current in the transient period, the dominant losses are the copper ones and consequently, only these losses are considered. One considers that the flux is constant and therefore, only one control current (torque producing current component in the case of the vector control) is used. The basic results for the optimal control of an electrical drive system in the above mentioned conditions are presented
**Keywords:** optimal control, electrical drive systems, speed dependent load torque, current control

## 1. INTRODUCTION

The aim of the optimal control [1,5], of the electrical drive systems is, in the most cases, the reducing of energy or power consumption [8], [9]. In the steady state operation, the control variables are chosen so that to be minimized the sum of the iron and Joule power losses [2,9]. The dynamic processes have an evolution in time, so that one has to be minimized the energy and not the power. In addition, in the most of these cases, the optimization is formulated in terms of the minimization only of the copper losses, since they significantly overcome the iron losses [2,11,15].

The optimal control problem for the electrical drive systems can differ depending on the type of the motor, on the terminal conditions, or on the possible presence of the restrictions. [4,11-14]. The use of the current or of the voltage control, or the use of one or two control variables have also an influence in the formulation and solving of the optimization problems. The mathematical model of the optimized system depends especially on the type of the drive motor [3,4,10,14]. If some assumptions are adopted, one can obtain a similar mathematical model for different motor types. The obtained results can be extended in this case for a large category of drive systems.

The present paper is a continuation of some previous authors' studies started with [3] and refers to the optimal control of the current controlled electrical drive systems [6]. An angular speed dependent load torque is considered in the present case. The optimization is done in the sense of minimization of the losses in the transient period. Taking into account the great values of the current in the transient period, the dominant losses are the copper ones and consequently, only these losses are considered. One considers that the flux is constant and therefore, only one control current (torque producing current component in the case of the vector control) is used. The basic results for the optimal control of an electrical drive system in the above mentioned conditions are presented in the next section. The sections 3 and 4 refer to some remarks and to the simulation results and the conclusions are finally presented.

## 2. MAIN RESULTS

One considers that the drive motor is current controlled and the flux is constant. Hence the electromagnetic torque can be expressed for the most types of the motors as

$$m(t) = c \cdot i(t) \tag{1}$$

where i is the current / the torque producing component of the control current and c is a constant of the motor. Also one considers the load torque linearly dependent on the angular speed ω:

$$m_r(t) = a \cdot \omega(t) + b \tag{2}$$

(a and b are known constants).

The moving equation for the drive system is

$$\dot{\omega}(t) = \gamma \cdot i(t) - \alpha \cdot \omega(t) - \beta\,, \tag{3}$$

with $\gamma = \frac{c}{J},\ \alpha = \frac{a}{J},\ \beta = \frac{b}{J}$, where J is inertia momentum of the drive system.

The optimization criterion refers to the Joule losses and one introduces the performance index

$$I = \frac{1}{2}\int_0^T i^2(t)dt \tag{4}$$

The optimal control problem refers to the system (3) and the criterion (4). The Hamiltonian of the problem is

$$H = \frac{1}{2}i^2(t) + \lambda(t)\cdot(\gamma \cdot i(t) - \alpha \cdot \omega(t) - \beta) \tag{5}$$

where λ(t) is the costate variable.

The necessary minimum condition ($\partial H/\partial i = 0$) leads to

$$\lambda(t) = -\frac{1}{\gamma}\cdot i(t) \tag{6}$$

It results from the Hamilton canonical equation ($\partial H/\partial \omega = -\dot{\lambda}(t)$)

$$\dot{\lambda}(t) = \alpha \cdot \lambda(t). \tag{7}$$

Taking into account the proportionality (6) between the variables i and λ, it results:

$$\dot{i}(t) = \alpha \cdot i(t) \tag{8}$$

The system given by canonical equations ((3) and (7)) is usually used to solve the problem. But, in this case, it is more convenient to use the system (3) and (8). The solution is obtained removing the variable i(t) from these equations. For this aim, one derives the equation (3) and replaces the derivative $\dot{i}(t)$ from (8), resulting

$$i(t) = \frac{1}{\gamma}(\dot{\omega}(t) + \frac{1}{\alpha}\ddot{\omega}(t)) \tag{9}$$

On the other hand, one obtains from (3)

$$i(t) = \frac{1}{\gamma}(\alpha \cdot \omega(t) + \beta + \dot{\omega}(t)) \tag{10}$$

It results from the last equations

$$\ddot{\omega}(t) - \alpha^2 \cdot \omega(t) - \alpha \cdot \beta = 0 \tag{11}$$

The solution to the equation (11) is

$$\omega(t) = C_1 \cdot e^{\alpha t} + C_2 \cdot e^{-\alpha t} - \frac{\beta}{\alpha} \tag{12}$$

where $C_1$ and $C_2$ are constants.

The angular acceleration is

$$\dot{\omega}(t) = \alpha \cdot (C_1 \cdot e^{\alpha t} - C_2 \cdot e^{-\alpha t}) \tag{13}$$

If the initial and final conditions are imposed

$$\omega(0) = 0 \quad \text{and} \quad \omega(T) = \omega_f, \tag{14}$$

it results

$$\begin{aligned} C_1 &= \frac{\omega_f}{\varepsilon} + \frac{1}{\varepsilon}\frac{\beta}{\alpha}(1 - e^{-\alpha T}) \\ C_2 &= \frac{\beta}{\alpha} - C_1 = \frac{\beta}{\alpha} - \frac{1}{\varepsilon}\frac{\beta}{\alpha}(1 - e^{-\alpha T}) - \frac{\omega_f}{\varepsilon} \end{aligned} \tag{15}$$

with $\varepsilon = e^{\alpha T} - e^{-\alpha T}$.

If $e^{-\alpha T} \approx 0$ it results $\varepsilon \approx e^{\alpha T}$ and then the constants have a simplified form:

$$C_1 \approx \frac{1}{e^{\alpha T}}(\omega_f + \frac{\beta}{\alpha}) \quad C_2 \approx \frac{\beta}{\alpha} - \frac{1}{e^{\alpha T}}(\omega_f + \frac{\beta}{\alpha}) \tag{16}$$

The control current results from (3), (12) and (13):

$$i(t) = \frac{2a}{c}\cdot C_1 \cdot e^{\alpha t} \tag{17}$$

or, if one takes into account the simplified form of the constant $C_1$,

$$i(t) \approx \frac{2a}{c}\cdot\frac{1}{e^{\alpha T}}(\omega_f + \frac{\beta}{\alpha})\cdot e^{\alpha t} = \frac{2}{c}\cdot m_{rf}\cdot e^{\alpha(t-T)} \tag{18}$$

where

$$m_{rf} = a \cdot \omega_f + b \tag{19}$$

is the value of the load torque at the final moment T.

The initial and final values of the current are:

$$i(0) = \frac{2a}{c}\cdot C_1 \approx \frac{2}{c}\cdot m_{rf}\cdot e^{-\alpha T} \quad \text{and} \quad i(T) = \frac{2a}{c}\cdot C_1 \cdot e^{\alpha T} \approx \frac{2}{c}\cdot m_{rf} \tag{20}$$

## 3. REMARKS

Using the relation (17), the dissipated energy can be expressed as

$$E = r\cdot\int_0^T i^2(t)dt = r\frac{4a^2}{c^2}C_1^2\cdot\int_0^T e^{2\alpha t}dt = r\frac{4\alpha^2}{\gamma^2}C_1^2\cdot\frac{1}{2\alpha}e^{2\alpha t}\Big|_0^T = r\frac{2\alpha}{\gamma^2}C_1^2\cdot(e^{2\alpha T} - 1) \tag{21}$$

or, in a simplified form based on (18),

$$E = r \cdot \int_0^T i^2(t)dt \approx \frac{4r}{c^2} m_{rf}^2 \cdot \int_0^T e^{2\alpha(t-T)}dt = \frac{4r}{c^2} m_{rf}^2 \cdot \frac{1}{2\alpha} e^{2\alpha(t-T)}\Big|_0^T = \frac{4r}{c^2} m_{rf}^2 \cdot \frac{1}{2\alpha}(1-e^{-2\alpha T}) \approx \frac{2r}{\alpha \cdot c^2} m_{rf}^2 \quad (22)$$

with $m_{rf}$ given by (19) and r is the resistance of the winding.
Using the notations introduced in (3), this last relation of the energy can be expressed as:

$$E \approx \frac{2r}{\alpha \cdot c^2}(a \cdot \omega_f + b)^2 \quad (23)$$

It is interesting to compare these minimal losses with the dissipated energy for another applied control current, for instance a constant current $i_0$. The solution to the equation (3) in this case is

$$\omega(t) = C' \cdot e^{-\alpha t} + \frac{\gamma}{\alpha} i_0 - \frac{\beta}{\alpha} \quad (24)$$

For the same initial condition $\omega(0) = 0$ , it results $C' = \frac{\beta}{\alpha} - \frac{\gamma}{\alpha} i_0$, so that

$$\omega(t) = \left(\frac{\gamma}{\alpha} i_0 - \frac{\beta}{\alpha}\right)(1 - e^{-\alpha t}) \quad (25)$$

The angular speed achieved in steady state is

$$\omega_f = \frac{\gamma}{\alpha} i_0 - \frac{\beta}{\alpha} \quad (26)$$

This speed is obtained for the current

$$i_0 = \frac{1}{\gamma}(\alpha \cdot \omega_f + \beta) \quad (27)$$

or

$$i_0 = \frac{1}{c}(a \cdot \omega_f + b) \quad (28)$$

The steady state is achieved approximately in the time $T' = 4/\alpha$ and the energy consumed in this time is

$$E' = r \cdot i_0^2 \cdot T' = \frac{4r}{\alpha \cdot c^2}(a \cdot \omega_f + b)^2 = \frac{4r}{\alpha \cdot c^2} m_{rf}^2 \quad (29)$$

If one compares this value with (22), it results a double value of the energy losses as in the optimal control case.
The torques that appear in the drive system are time variant. Using (12) and (13), it results:

- the load torque:

$$m_r(t) = a \cdot \left(C_1 \cdot e^{\alpha t} + C_2 \cdot e^{-\alpha t} - \frac{\beta}{\alpha}\right) + b = a \cdot (C_1 \cdot e^{\alpha t} + C_2 \cdot e^{-\alpha t}) \quad (30)$$

- the dynamic torque:

$$m_d = J \cdot \dot{\omega} = a \cdot (C_1 \cdot e^{\alpha t} - C_2 \cdot e^{-\alpha t}) \quad (31)$$

- the electromagnetic torque:

$$m(t) = m_r + m_d = 2a \cdot C_1 \cdot e^{\alpha t}, \quad (32)$$

in concordance with (17).
The terminal conditions (14) are fixed in the solved optimal control problem. There are also other possibilities for specification of the terminal conditions. The case when final state $\omega_f = \omega(T)$ is imposed, but the final time T is free (the initial condition remain the same as in (14)) is further analysed.
A transversality condition is introduced in this case. Since the performance index does not explicitly contain T, the mentioned condition is in the form $H(\omega(T), i(T), \lambda(T)) = 0$, that is, from (5):

$$H = \frac{1}{2} i^2(T) + \lambda(T) \cdot (\gamma \cdot i(T) - \alpha \cdot \omega(T) - \beta) = 0 \quad (33)$$

It results from (33):

$$i_f = i(T) = \frac{2}{c} \cdot m_{rf}, \quad (34)$$

with $m_{rf}$ given by (19).
The solution to the equation (8) is

$$i(t) = C_0 \cdot e^{\alpha t} \quad (35)$$

The constant $C_0$ results using the final condition (34) and one obtains

$$i(t)=\frac{2}{c}\cdot m_{rf}\cdot e^{\alpha(t-T)} \tag{36}$$

In this case, the solution to the equation (3) is in the form

$$\omega(t)=k\cdot e^{\alpha t}-\frac{\beta}{\alpha} \tag{37}$$

and the initial condition (14) leads to

$$\omega(t)=\frac{\beta}{\alpha}\cdot(e^{\alpha t}-1) \tag{38}$$

It results from (38) that the value of the final time in this case is

$$T=\frac{1}{\alpha}\ln\frac{\alpha\cdot\omega_f+\beta}{\beta} \tag{39}$$

The obtained solution (38) corresponds to $C_2=0$ and $C_1=\beta/\alpha$ in the previous variant.
It results in this case from (17) the current

$$i(t)=\frac{2b}{c}e^{\alpha t} \tag{40}$$

- the load torque is

$$m_r(t)=a\cdot\omega(t)+b=b\cdot e^{\alpha t} \tag{41}$$

- the dynamic torque is

$$m_d=J\cdot\dot{\omega}=b\cdot e^{\alpha t} \tag{42}$$

- the electromagnetic torque is

$$m(t)=m_r+m_d=2b\cdot e^{\alpha t}, \tag{43}$$

in concordance with (40).
The property indicated by the last relations is: in the optimal control with free final time, the dynamic and load torque are equal, that is, the optimal electromagnetic torque is double as the load one. A similar property was indicated in [3] and other authors' papers [3],[4],[6] for constant or variable load torque, but not depending on angular speed.
For the optimal control problem with fixed initial and final conditions, these properties results from the previous equations only for the final time T.
The equations (38) and (40) indicate a linear dependence between i(t) and ω(t), and this fact suggests a simple modality for obtaining a feedback control.

## 4. SIMULATION RESULTS

In order to visualize the dynamic variables in the transient period, some numerical simulations, using various types of electrical machines (dc/ac), were done. The general simulated structure of the optimal control system is presented in Figure 1.

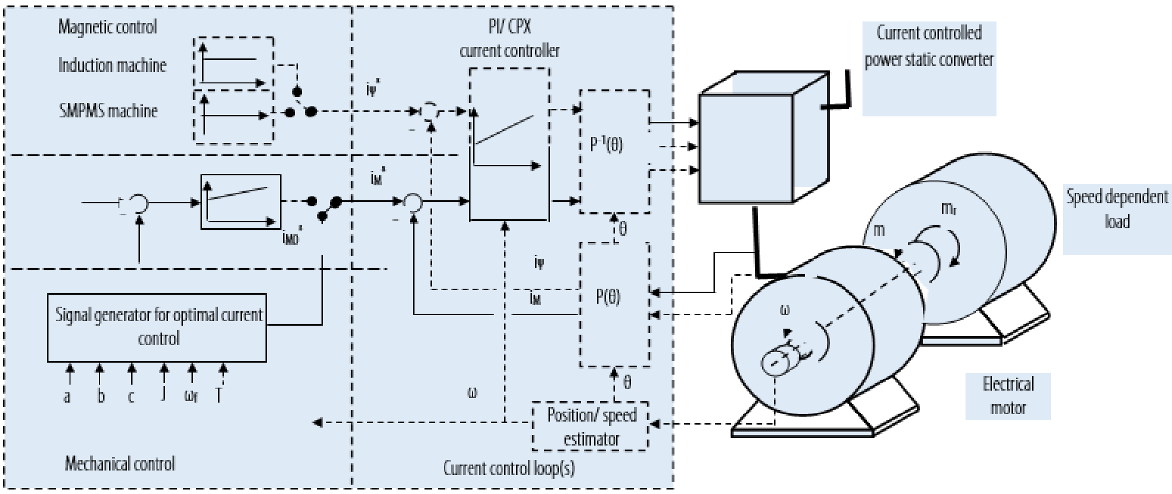


Figure 1. The general simulated structure of the optimal control system

Basically, it comprises a current control loop(s), which transforms the voltage source static converter (chopper or inverter) into a current source one. When it comes to AC electrical machine (induction machine – IM, or surface mounted permanent synchronous machine – SMPMSM) some supplementary signal processing blocks (dotted blocks and signals) are needed in order to ensure the control frame orientation for field oriented control, and the magnetic control of the ac electrical machine. The stator magnetic flux of dc electrical machine is

ensured by permanent magnets and, hence, the only controlled current is the torque producing one ($i_M$), using a classical PI structure. On the other hand, the currents of the ac machines (stator flux producing current, $i_\Psi$, and torque producing current, $i_M$) are controlled using a unified complex vector current controller – CPX [7].
In the classical master-slave control structures, other kind of controllers (speed, position, pressure) might exist upstream of these current control loops. But a specific signal (reference) generator should be added for this optimal control, in order to generate the torque producing current trajectory (profile), $i_{MO}$, as in Figure 1.
As already was mentioned, the proposed optimal algorithm for drive systems with speed dependent load torque was tested using the electrical machine having the characteristics and parameters presented in Table 1.

Table 1. The characteristics and the parameters of electrical drive systems

| Type of electrical machine | $P_N$ [kW] | $U_N$ [V] | $I_N$ [A] | $\omega_N$ [rpm] | $n_p$ [%] | $m_N$ [Nm] | $\Psi_N$ [Wb] | $r_{SN}$ [Ω] | $r_{RN}$ [Ω] | $L_{SN}$ [mH] | $L_{RN}$ [mH] | $L_{MN}$ [mH] | $J_N$ [kgm$^2$] | $D_N$ [Nms/rad] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| PM dc machine | 3 | 220 | 17.5 | 1200 | 2 | 23 | 0.77 | - | 1.43 | - | 29.8 | - | 0.5 | 0.025 |
| SMPMS machine | 2 | 190 | 4.1 | 4500 | 3 | 4.24 | 0.235 | 1.68 | - | 7.89 | - | - | 0.051 | 0.0025 |
| Induction machine | 1.5 | 220 | 3.8 | 1410 | 2 | 10.15 | - | 3.5 | 3.24 | 280 | 280.9 | 265.5 | 0.0212 | 0.0054 |

The parameters taken into account for generation of the torque producing current trajectory, described by (17), are presented in Table 2.

Table 2. The parameters needed by torque producing current generator

| Type of electrical machine | a | b | c | α | β | γ | $\omega_f$ [rad/s] | T [s] | $m_{rf}$ [Nm] |
|---|---|---|---|---|---|---|---|---|---|
| PM dc machine | 0.127 | 1.00 | 1.547 | 0.254 | 2 | 3.094 | 125 | 4 | 16.87 |
| SMPMS machine | 0.0148 | 0.05 | 1.0597 | 2.4183 | 9.8039 | 20.77 | 200 | 4 | 3.0157 |
| Induction machine | 0.025 | 0.05 | 1.08 | 0.9233 | 0.2358 | 50.94 | 140 | 4 | 3.50 |

For all three electrical drive systems there was imposed the same transient period, i.e. 4s, but different final angular speeds, in order to study only the cases where the fluxes are constant. Table 3 summarizes the predicted quantities based on the analytical approach (initial/final values of optimal torque producing current – (20), dissipated energy – (21)), and those obtained by numerical simulations.

Table 3. Analytical/numerical solution of some quantities

| Type of electrical machine | $i_{MO}(0)$ [A] | | $i_{MO}(T)$ [A] | | $\omega_f$ [rad/s] | | Energy [J] | |
|---|---|---|---|---|---|---|---|---|
| | pred. | sim. | pred. | sim. | imposed | sim | pred. | sim. |
| PM dc machine | 8.89 | 8.9 | 24.56 | 24.56 | 125 | 124.7 | 1476.4 | 1469.6 |
| SMPMS machine | 1.96 | 2 | 6.27 | 6.3 | 200 | 199.9 | 102.66 | 102.6 |
| Induction machine | 0.058 | 0.1 | 6.68 | 6.6 | 140 | 132 | 62.46 | 62.3 |

In order to calculate these quantities, the exact analytical relations were used, i.e. (17) and (21), and not the approximated ones, i.e. (18) and (22). In this way, one can notice that there is a good agreement between the predicted quantities and those obtained by numerical simulations. Also the final reached angular speeds are very close to the imposed final values.
Some relevant traces regarding the transient period of the analysed electrical drive systems are presented in the next figures. For the electrical drive system with PM dc machine only the torque producing current (rotor current) is presented. On the other hand, for the electrical drive systems using ac machines, both flux producing current and torque producing current are presented. The next plot presents the time-dependent evolution of the developed electromagnetic torque and the speed dependent load torque. The next plot presents the variation of the speed of the electrical drive system. Finally, the last plot of each figure visualises the evolution of the dissipated energy, estimated during the simulation of the transient period.
The first plot of Figure 2 presents the optimal torque producing current, generated by the signal generator, and the controlled current, for the analysed electrical drive system based on PM dc electrical machine. There is a very good coincidence between these currents; only a very small difference is observable at the beginning of the transient process. A similar remark is valuable for the next figures. The used algorithm for generating this reference current is based on relation (17).
The same traces are presented in Figure 3, for the same electrical drive system, but for free final time problem. In this case the signal generator is based on relation (40). For the same final angular speed, i.e. 125 rad/s, the final time value (39) is estimated having the value 11.12s. The same value was obtained by numerical simulation. Regarding the dissipated energy by Joule losses, the simulated final value is 1338.2 J, a little bit smaller than the previous one (1476.6 J).
Figures 4 and 5 present the results obtained in transient period for electrical drive systems using AC electrical machines. The optimal control in these cases refers only to the problems with the fixed final time.

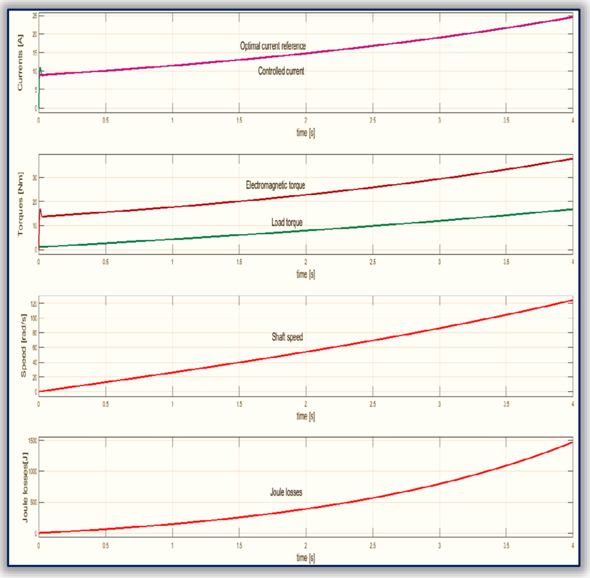


Figure 2. Optimal control of PM dc machine based on imposed final time

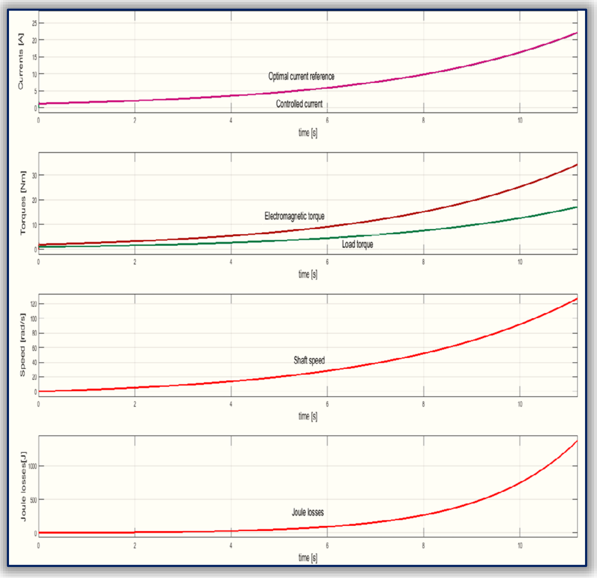


Figure 3. Optimal control of PM dc machine based on free final time

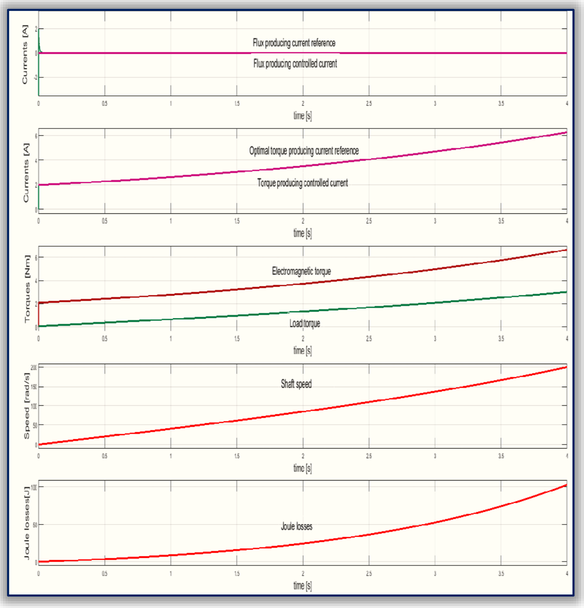


Figure 4.Optimal control of SMPMS machine \based on imposed final time

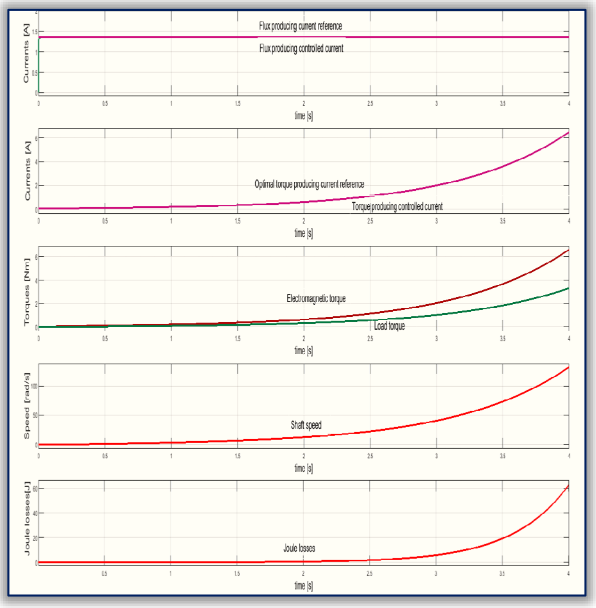


Figure 5. Optimal control of induction machine based on imposed final time

Figure 4 presents the dynamics of the SMPMSM electrical drive system, while Figure 5 presents the dynamics of the electrical drive system based on induction machine. As already was mentioned, in addition to the plots of Figure 2 and Figure 3, on the top of each figure there are also presented the references of the flux producing current and the controlled one.

Finally, one can remark also that, irrespective of the machine type, the currents control loops work very well.

## 5. CONCLUSIONS

The paper deals with the optimal control of the electrical drive systems from energetic point of view. The current control is considered for a large category of motor types. The study considers the case of the load torque linear dependent on angular speed.

The main relations for the problem with fixed terminal conditions are established. Comparing with the case of the start of the motor with a constant current, it results a dissipated energy two times smaller in the optimal control case.

The problem with free final time is also analysed. A linearity between current and speed appears in this case and it is a convenient situation because it allows introducing a feedback control. It was established an interesting property: for the same case the optimal electromagnetic torque is double as the load torque. A similar result was obtained by authors in previous papers for constant or time variant load torque, but not depending on speed.

The simulation results for different types of motors are presented.
The intention of the authors is to extend the research to other related cases as: the feedback optimal control, the case of the restricted current, the case of quadratic dependence of speed on the load torque.

**Note**: This paper was presented at XX$^{th}$ National Conference on Electric Drives – CNAE 2021/2022, organized by the Romanian Electric Drive Association and the Faculty of Electrotechnics and Electroenergetics –University Politechnica Timisoara, in Timisoara (ROMANIA), between May 12–14, 2022 (initially scheduled for October 14–16, 2021).

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