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STEADY-STATE HEAT CONDUCTION PROBLEMS FOR NON-HOMOGENEOUS HOLLOW ELLIPTICAL TWO-DIMENSIONAL DOMAIN

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Abstract: The paper deals with a two-dimensional boundary value problem of steady-state heat conduction in non-homogeneous hollow elliptical domain. First one is the layered elliptical domain, the thermal conductance in each elliptical rings is constant. The layered non-homogeneous domain is considered as a union of elliptical rings which have different material properties. The boundary curves of elliptical rings are confocal ellipses. In the second case, the functionally graded type of non-homogeneity is considered, the thermal conductance is a smooth function of a curvilinear coordinate. All results of the paper are based on Fourier's theory of heat conduction in non-homogeneous solid bodies.
Keywords: heat conduction, steady-state, layered elliptical domain, functionally graded

1. INTRODUCTION

Numerous studies and textbooks of heat transfer deal with the steady state thermal conduction in non-homogeneous solid bodies [1-6]. In this paper a two-dimensional boundary-value problem of steady-state heat conduction in non-homogeneous hollow elliptical domain is considered. The boundary curves of the hollow elliptical domain are confocal ellipsis. Bagda and Khobragade [6] determine the temperature distribution in a finite elliptical cylinder with the help of Mathien transform and Marchi-Fasulo transform methods. They considered a nonsteady-state heat conduction problem. Another paper of Bagda and Khobragade [12] presents the theoretical treatment of the temperature field in a hollow elliptical cylinder due to partially distributed heat supply on the outer elliptical boundary surface. Integral transform techniques have been used to obtain the solution of the considered steady-state heat conduction problem. Heat conduction in elliptical cylinders and cylindrical shells is studied by Dicker and Friedman [9]. They obtained the solution by means of Galerkin method, used together with the Laplace transform for the transient temperature distribution in elliptical domain and cylindrical shell. A one-term and three-term approximations are employed for the ellipse and a one-term approximation of cylindrical shell. Lopata et. al., [10] calculated the values of heat transfer coefficient from the wall of the elliptical pipe to the water flowing inside were determined using the data from conducted measurements under the condition of constant heat flux. In paper by Khan et. al., [11] an analytical solution is given for a transient heat conduction problem for confocal elliptical region using elliptical curvilinear coordinates. The obtained solution for temperature field was applied to solve a plane thermal stress problem.

In this paper a two-dimensional stead-state heat conduction problem in non-homogeneous hollow elliptical domain is considered. Two types of non-homogeneity are studied. The first one is a layered elliptical domain in which the thermal conductance is piecewise constant function of a curvilinear coordinate and the second one is a continuous function of one curvilinear coordinate which describe the boundary contour of the considered hollow elliptical region.

2. CURVILINEAR COORDINATES

Figure 1 shows the hollow elliptical domain A, whose boundary curves $\partial \mathrm{A}_1$ and $\partial \mathrm{A}_2$ are confocal ellipses. The common focuses of boundary ellipses are point F_1, F_2 (see Figure 1).

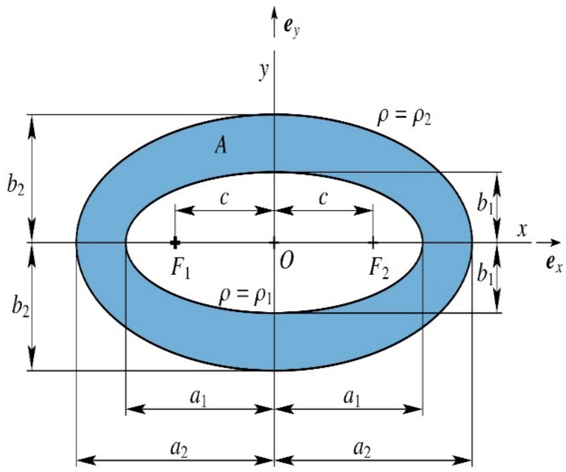

Figure 1. Hollow elliptical domain

The following equation is valid

$$\mathrm{c}^2 = \mathrm{a}_1^2 - \mathrm{b}_1^2 = \mathrm{a}_2^2 - \mathrm{b}_2^2, \qquad \overline{\mathrm{F}_1\mathrm{F}_2} = 2\mathrm{c}. \tag{1}$$

To formulate the steady-state heat conduction problem it is necessary to introduce an orthogonal curvilinear coordinate system. The definition of curvilinear coordinates ρ, α is given by the following equations

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$$x = \left(\rho + \frac{c^2}{4\rho}\right)\cos\alpha\,,\ \ y = \left(\rho - \frac{c^2}{4\rho}\right)\sin\alpha\,,\ \ \rho_1 \le \rho \le \rho_2\ \ 0 \le \alpha \le 2\pi. \tag{2}$$

Equation of boundary curve $\partial A_i\ (i = 1,2)$ in curvilinear coordinates ρ and α is

$$\rho\ =\ \rho_i\ =\ \text{const.}\quad 0\ \le\ \alpha\ \le\ 2\,\pi. \tag{3}$$

From Eqs. (2) and (3) it follows that the semi axes of the boundary ellipses are

$$a_i = \sqrt{\rho_i^2 + \frac{c^2}{2} + \frac{c^4}{16\rho_i^2}},\ \ b_i = \sqrt{\rho_i^2 - \frac{c^2}{2} + \frac{c^4}{16\rho_i^2}}\ \ (i = 1,2). \tag{4}$$

The unit vectors of curvilinear coordinate system (ρ, α) are [7]

$$\mathbf{e}_\rho = \frac{1}{\sqrt{\left(\frac{\partial x}{\partial \rho}\right)^2 + \left(\frac{\partial y}{\partial \rho}\right)^2}}\left(\frac{\partial x}{\partial \rho}\mathbf{e}_x + \frac{\partial y}{\partial \rho}\mathbf{e}_y\right) = \frac{1}{H_\rho}\left[\left(1 - \frac{c^2}{4\rho^2}\right)\cos\alpha\mathbf{e}_x + \left(1 + \frac{c^2}{4\rho^2}\right)\sin\alpha\right]\mathbf{e}_y, \tag{5}$$

where

$$H_\rho^2 = 1 + \frac{c^4}{16\rho^4} - \frac{c^2}{2\rho^2}\cos 2\,\alpha, \tag{6}$$

$$\mathbf{e}_\alpha = \frac{1}{\sqrt{\left(\frac{\partial x}{\partial \alpha}\right)^2 + \left(\frac{\partial y}{\partial \alpha}\right)^2}}\left(\frac{\partial x}{\partial \alpha}\mathbf{e}_x + \frac{\partial y}{\partial \alpha}\mathbf{e}_y\right) = \frac{1}{H_\alpha}\left[-\rho\left(1 + \frac{c^2}{4\rho^2}\right)\sin\alpha\mathbf{e}_x + \rho\left(1 - \frac{c^2}{4\rho^2}\right)\cos\alpha\mathbf{e}_y\right], \tag{7}$$

where

$$H_\alpha^2 = \rho^2 H_\rho^2. \tag{8}$$

The curvilinear coordinate system ρ, α is an orthogonal coordinate system, since

$$\mathbf{e}_\rho \cdot \mathbf{e}_\alpha = 0 \quad \rho_1 \le \rho \le \rho_2 \quad \alpha_1 \le \alpha \le \alpha_2. \tag{9}$$

On the line element of $\alpha =$ const. curve is

$$ds_\alpha = H_\rho d\rho, \tag{10}$$

and on the line element of $\rho =$ const. curve is

$$ds_\rho = H_\alpha d\alpha = \rho H_\rho d\alpha. \tag{11}$$

The expression of area element dA in the $(\rho,\ \alpha)$ curvilinear coordinate system is as follows [7]

$$dA = H_\rho H_\alpha d\rho d\alpha = \rho H_\rho^2 d\rho d\alpha. \tag{12}$$

Let $F = F(\rho,\ \alpha)$ be an arbitrary differentiable function of its arguments ρ and α. The gradient of $F = F(\rho,\ \alpha)$ in curvilinear coordinates ρ and α can be represented as [7]

$$\nabla F = \frac{1}{H_\rho}\frac{\partial F}{\partial \rho}\mathbf{e}_\rho + \frac{1}{H_\alpha}\frac{\partial F}{\partial \alpha}\mathbf{e}_\alpha. \tag{13}$$

The divergence of the two-dimensional vector field $\mathbf{f} = f_\rho(\alpha,\ \rho)\mathbf{e}_\rho + f_\alpha(\alpha,\ \rho)\mathbf{e}_\alpha$ can be expressed as

$$\nabla \cdot \mathbf{f} = \frac{1}{H_\rho H_\alpha}\left[\frac{\partial}{\partial \rho}(H_\alpha f_\rho) + \frac{\partial}{\partial \alpha}(H_\rho f_\alpha)\right]. \tag{14}$$

In Eqs. (13) and (14)

$$\nabla = \frac{1}{H_\rho}\frac{\partial}{\partial \rho}\mathbf{e}_\rho + \frac{1}{H_\alpha}\frac{\partial}{\partial \alpha}\mathbf{e}_\alpha \tag{15}$$

is the del operator and the dot between two vectors in Eq. (14) denotes the scalar product.

3. FORMULATION OF THE HEAT CONDUCTION BOUNDARY VALUE PROBLEM

According to the Fourier's theory of heat conduction in solid body we have next equations

$$\mathbf{q} = -k\nabla T, \qquad \nabla \cdot \mathbf{q} = 0, \tag{16}$$

where $T = T(\rho,\ \alpha)$ is the temperature field, $\mathbf{q} = q_\rho(\rho,\ \alpha)\mathbf{e}_\rho + q_\alpha(\rho,\ \alpha)\mathbf{e}_\alpha$ is the heat flux vector and $k = k(\rho,\ \alpha)$ is the heat conduction coefficient. In Eq. (16) it is assumed that there are no internal heat sources. The boundary temperature is prescribed, that is we have

$$T(\rho_1,\ \alpha) = T_1, \quad T(\rho_2,\ \alpha) = T_2, \quad 0 \le \alpha \le 2\pi. \tag{17}$$

It is assumed that the temperature field and the thermal conductance depend only on the curvilinear coordinate ρ. In this case from Eq. (16) it follows that

$$\mathbf{q} = -k(\rho)\frac{1}{H_\rho}\frac{dT}{d\rho}\mathbf{e}_\rho. \tag{18}$$

Application of Eq. (14) gives

$$\frac{d}{d\rho}\left(\rho k(\rho)\frac{dT}{d\rho}\right) = 0 \qquad \rho_1 \le \rho \le \rho_2. \tag{19}$$

The solution of the boundary value problem formulated in Eqs. (18) and (19) can be represented as

$$T(\rho) = T_1 + (T_2 - T_1)\frac{\int_{\rho_1}^{\rho}\frac{d\rho}{\rho k(\rho)}}{\int_{\rho_1}^{\rho_2}\frac{d\rho}{\rho k(\rho)}} \qquad \rho_1 \le \rho \le \rho_2. \tag{20}$$

The overall heat transfer coefficient in steady state heat conduction problem is an important structural property of a solid body in which the heat is flowing between two separated parts of its boundary surfaces. From the higher temperature boundary part of the solid body to the lower temperature boundary part of

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the solid body the process of heat flow is characterized by the overall heat transfer coefficient Λ, whose definition is given by Eq. (21)

$$Q = \Lambda(T_1 - T_2) \qquad T_1 > T_2, \tag{21}$$

where Q is the heat flow in unit time and T_1 and T_2 are the boundary temperatures and Λ is the overall heat transfer coefficient. Expression of Q can be computed according to Eq. (22)

$$Q = -\int_{\partial A_2} \left(\frac{k(\rho)}{H_\rho}\frac{dT}{d\rho}\right)_{\rho=\rho_2} ds_\alpha = (T_1 - T_2)\frac{2\pi}{\int_{\rho_1}^{\rho_2}\frac{d\rho}{\rho k(\rho)}}. \tag{22}$$

For homogeneous elliptical tube from Eqs. (21) and (22) we obtain

$$\Lambda = \frac{2\pi k}{\ln\frac{\rho_2}{\rho_1}} \qquad k = \text{const.} \tag{23}$$

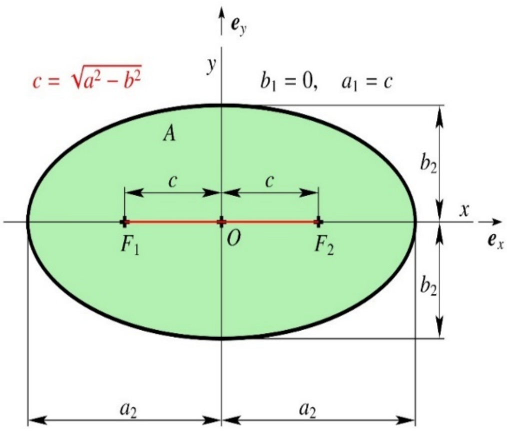

Figure 2. Ribbon insert in elliptical cylinder

Combination of Eq. (14) with (23) gives the expression of overall thermal transfer coefficient in terms of a_i, b_i as

$$\Lambda = \frac{2\pi k}{\ln\left|\frac{a_2+b_2}{a_1+b_1}\right|}, \qquad k = \text{const.} \tag{24}$$

In the case of degenerate internal boundary elliptical curve, when

$$a_1 = c, \qquad b_1 = 0, \qquad a_2^2 - b_2^2 = c^2, \tag{25}$$

since we have

$$\rho_i = \frac{1}{2}(a_i + b_i) \qquad i = 1,2. \tag{26}$$

The formula of overall thermal coefficient can be represented as (Figure 2)

$$\Lambda = \frac{2\pi k}{\ln\left|\frac{a_2+b_2}{c}\right|}. \qquad 27)$$

4. LAYERED HOLLOW ELLIPTICAL CROSS SECTION

Figure 3 shows a layered hollow elliptical cross section. A typical layer of the cross section is bounded by two ellipses whose equations in curvilinear coordinates ρ and α are

$$\rho = \rho_i = \text{const.} \qquad \rho = \rho_{i+1} = \text{const.} \qquad 0 \le \alpha \le 2\alpha. \tag{28}$$

Thermal conductance is constant on each layer, but it has different values, that is

$$k(\rho) = k_i = \text{const.} \qquad \rho_{i-1} < \rho < \rho_i \qquad 0 \le \alpha \le 2\pi \qquad (i = 1,2, \ldots n). \tag{29}$$

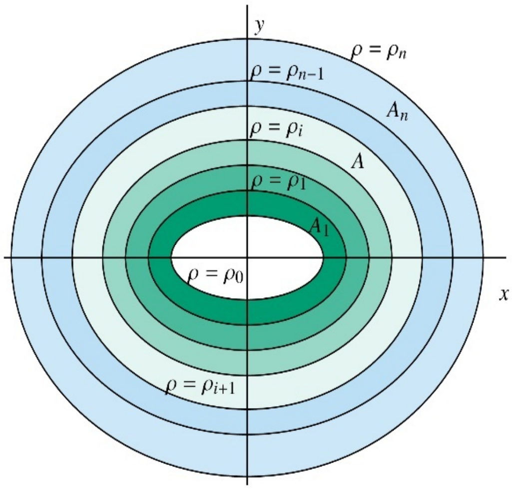

Figure 3. Layered elliptical plane domain

For a typical layer the solution of the heat conduction equation according to Section 3 of this paper can be represented as

$$T(\rho) = T_{i-1} + (T_i - T_{i-1})\frac{\ln\frac{\rho}{\rho_{i-1}}}{\ln\frac{\rho_i}{\rho_{i-1}}} \qquad \rho_{i-1} \le \rho \le \rho_i, \quad 0 \le \alpha \le 2\pi, \tag{30}$$

where

$$T_{i-1} = T(\rho_{i-1}), \qquad T_i = T_i(\rho_i) \qquad (i = 1,2, \ldots n) \tag{31}$$

are the unknown temperatures of the boundary curves of considered elliptical ring. The continuity condition for the heat flow at the common boundary of layer A_i and A_{i+1} can be formulated as

$$a_i T_{i+1} - (a_i + b_i)T_i + b_i T_{i-1} = 0 \qquad (i = 1,2, \ldots n-1), \tag{32}$$

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$$a_i = \frac{k_{i+1}}{\ln\frac{\rho_{i+1}}{\rho_i}}, \qquad b_i = \frac{k_i}{\ln\frac{\rho_i}{\rho_{i-1}}}, \qquad (i = 1,2, \dots n). \tag{33}$$

The boundary temperatures on the inner and outer boundary curves of the layered elliptical tube are prescribed

$$T(\rho_0) = T_0, \qquad T(\rho_n) = T_n. \tag{34}$$

The solution of the system of linear Eqs. (34) for $T_1, T_2, \dots T_{n-1}$ gives the values of unknown boundary temperatures of the component elliptical rings. The heat flown in the unit time on elliptical ring A_i can be computed as

$$Q_i = \Lambda_i(T_{i-1} - T_i) \qquad (i = 1,2, \dots n), \tag{35}$$

where

$$\Lambda_i = \frac{2\pi k_i}{\ln\frac{\rho_i}{\rho_{i-1}}} \qquad (i = 1,2, \dots n). \tag{36}$$

It is evident that Q_i does not depend on i, that is $Q_i = Q =$ const. The validity of Eq. (36) follows from Eq. (24). We introduce the concept of thermal resistance R_i as

$$R_i = \frac{1}{\Lambda_i}, \qquad (i = 1,2, \dots n). \tag{37}$$

The validity of the following equations follows from Eq. (36) and the definition of R_i

$$\begin{gathered} T_0 - T_1 = QR_1 \\ T_1 - T_2 = QR_2 \\ \dots \\ T_{n-1} - T_n = QR_n. \end{gathered} \tag{38}$$

By adding the equations of the group (38) we get that

$$T_0 - T_n = Q(R_1 + R_2 + \dots + R_n) = Q\left(\frac{1}{\Lambda_1} + \frac{1}{\Lambda_2} + \dots + \frac{1}{\Lambda_n}\right). \tag{39}$$

Eq.(39) shows that the thermal resistance of the connected elliptical rings as a whole body is

$$R = R_1 + R_2 + \dots + R_n \tag{40}$$

and we have for the overall heat transfer coefficient of the composite hollow elliptical domain

$$\frac{1}{\Lambda} = \frac{1}{\Lambda_1} + \frac{1}{\Lambda_2} + \dots + \frac{1}{\Lambda_n}. \tag{41}$$

5. NUMERICAL EXAMPLES

— Layered elliptical tube

The geometrical data of the five-layer hollow elliptical domain are as follows

$$\rho_0 = 0.03\,\text{m}, \ \ \rho_1 = 0.04\,\text{m}, \ \ \rho_2 = 0.05\,\text{m}, \ \ \rho_3 = 0.06\,\text{m}, \ \ \rho_4 = 0.07\,\text{m}, \ \ \rho_5 = 0.08\,\text{m}. \tag{42}$$

The values of thermal conductance are

$$k_1 = 50\,\frac{\text{W}}{\text{mK}}, \ \ k_2 = 237\,\frac{\text{W}}{\text{mK}}, \ \ k_3 = 80\,\frac{\text{W}}{\text{mK}}, \ \ k_4 = 35\,\frac{\text{W}}{\text{mK}}, \ \ k_5 = 65\,\frac{\text{W}}{\text{mK}}. \tag{43}$$

The boundary temperatures are

$$T_0 = T(\rho_0) = 450\,\text{K}, \qquad T_5 = T(\rho_5) = 250\,\text{K}. \tag{44}$$

The temperature distribution as a function of the curvilinear coordinate is given by the next function

$$T(\rho) = \big(H(\rho-\rho_0) - H(\rho-\rho_1)\big)\left(T_0 + \frac{T_1 - T_0}{\ln\frac{\rho_1}{\rho_0}}\ln\frac{\rho}{\rho_0}\right) + \big(H(\rho-\rho_1) - H(\rho-\rho_2)\big)\left(T_1 + \frac{T_2 - T_1}{\ln\frac{\rho_2}{\rho_1}}\ln\frac{\rho}{\rho_1}\right) + \big(H(\rho-\rho_2) - H(\rho-\rho_3)\big)\left(T_2 + \frac{T_3 - T_2}{\ln\frac{\rho_3}{\rho_2}}\ln\frac{\rho}{\rho_2}\right)\big(H(\rho-\rho_3) - H(\rho-\rho_4)\big)\left(T_3 + \frac{T_4 - T_3}{\ln\frac{\rho_4}{\rho_3}}\ln\frac{\rho}{\rho_3}\right) + \big(H(\rho-\rho_4)\big)\left(T_4 + \frac{T_5 - T_4}{\ln\frac{\rho_5}{\rho_4}}\ln\frac{\rho}{\rho_4}\right) \tag{44}$$

In Eq. (45) $H = H(\rho)$ is the Heaviside function [7]

$$H(\rho - a) = \begin{cases} 0 & \rho < a, \\ 1 & \rho > a. \end{cases} \tag{46}$$

The graph of $T = T(\rho)$ as a function of ρ is shown in Figure 4.

The application of the formula (40) gives the result for the thermal resistance

$$R = 0.002\,456\,211\,03\,\frac{\text{mK}}{\text{W}}. \tag{47}$$

— Elliptical tube made of functionally graded material

The following data are used in this example

$$c = 0.0025\,\text{m}, \ \ a_1 = 0.003\,\text{m}, \ \ b_1 = 0.001\,658\,312\,395\,\text{m}, \ \ a_2 = 0.007\,\text{m}, \ \ k_0 = 50\,\frac{\text{W}}{\text{mK}}, \\ b_2 = 0.006\,538\,348\,415\,\text{m}, \ \ \rho_1 = 0.002\,329\,156\,198\,\text{m}, \ \ \rho_2 = 0.006\,769\,174\,208\,\text{m}, \tag{48}$$

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$$k(\rho) = k_0\left(\frac{\rho}{\rho_2}\right)^n. \tag{49}$$

In Eq. (49), $\mathbf{n}$ is a material parameter so-called power index.
The application of Eq. (19) yields to the expression of the temperature field

$$T(\rho, n) = T_1 + (T_2 - T_1)\frac{\left(\frac{\rho_2}{\rho}\right)^n - \left(\frac{\rho_1}{\rho}\right)^n}{1 - \left(\frac{\rho_2}{\rho_1}\right)^n} \qquad \rho_1 \le \rho \le \rho_2. \tag{50}$$

The plot of $T = T(\rho, n)$ for the five different values of power index n is shown in Figure 5.

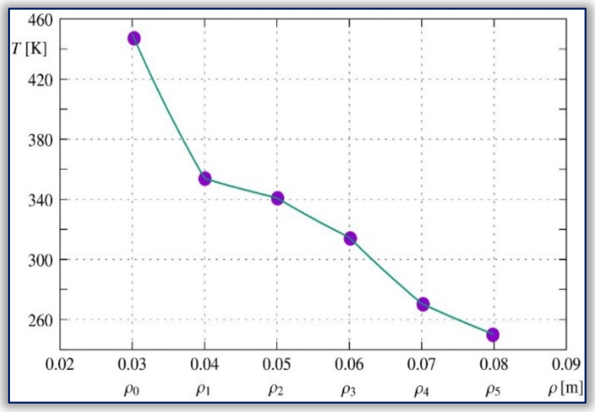

Figure 4. The plot of the temperature as a function of ρ.

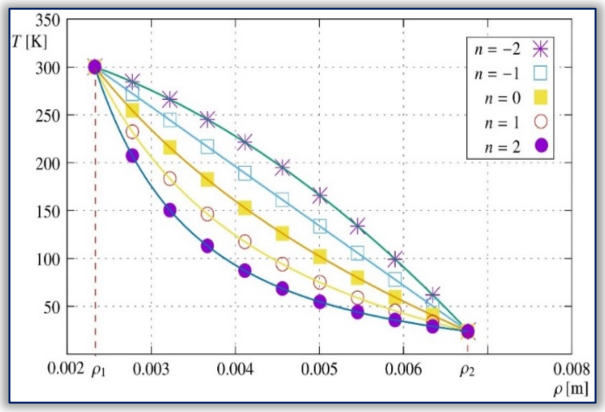

Figure 5. The plot of the temperature function for five different values of power index n $(n = -2, -1, 0, 1, 2)$.

The thermal resistance $R = R(n)$ can be computed from the following formula

$$R(n) = \frac{\left(\frac{\rho_1}{\rho_2}\right)^{-n} - 1}{2\pi n k_0}. \tag{51}$$

The graph of $R = R(n)$ as a function of power index is given Figure 6, for $-3 \le n \le 3$.

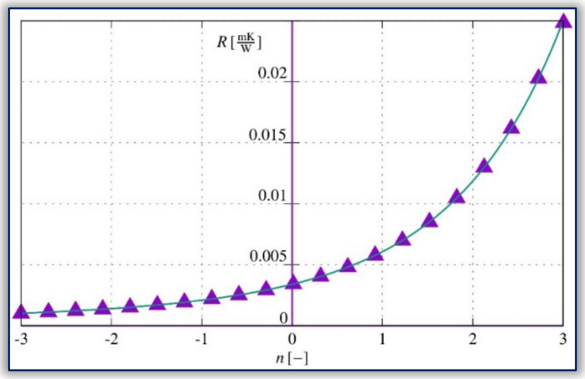

Figure 6. The graph of the function $R = R(n)$ for $-3 \le n \le 3$.

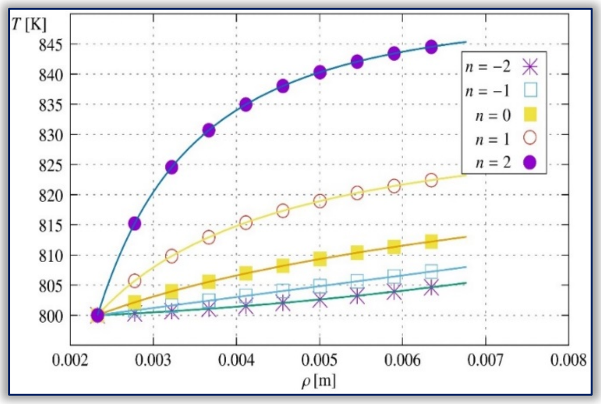

Figure 7. The graphs of temperature function for five different value of power index in Example 3 $(-2 \le n \le 2)$.

— Elliptical tube with prescribed heat input

In this example the following boundary conditions are prescribed for the hollow elliptical domain

$$T(\rho_1) = T_1 \qquad 0 \le \alpha \le 2\pi, \tag{52}$$

$$-k(\rho)\frac{1}{H_\rho}\frac{dT}{d\rho} = \frac{q}{H_\rho} \qquad \rho = \rho_2 \qquad 0 \le \alpha \le 2\pi. \tag{53}$$

In Eq. (52) $\mathbf{q}$ is a given constant. The thermal conductance and other data are the same as in Example 2, expect the value of T_1. The temperature of the outer boundary ellipse is obtained from the solution of heat conduction equation under the boundary conditions (52) and (53). It is easy to prove that the solution of the heat conduction equation for functionally graded material when $k = k(\rho)$ is a given smooth function, is as follows

$$T(\rho) = T_1 + q\rho_2\int_{\rho_1}^{\rho}\frac{dp}{pk(p)}. \tag{54}$$

The solutions of Eq. (54) for

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$$T_1 = 800\,\mathrm{K} \qquad q = -9 \times 10^4 \,\frac{\mathrm{W}}{\mathrm{m}^2} \tag{55}$$

as a function of power index n and the curvilinear coordinate ρ are shown in Figure 7.

6. CONCLUSIONS

A two-dimensional boundary value problem of steady-state heat conduction in non-homogeneous hollow elliptical region is considered.

Two types of non-homogeneity are analyzed. The first one is a layered elliptical domain which consists of several elliptical rings. The second one is a functionally graded non-homogeneity when the material parameter is a continuous function of a curvilinear coordinate. The presented analytical solutions can be used in the solution of plane thermoelastic problems when the temperature field can be determined independently of deformation. Another possible application of the numerical results of this paper is to verify the accuracy of the usual numerical methods such as FEM, finite difference method, boundary element method, etc.

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