[1.] O. O. OLUBANWO, [1.] S. A. ONITILO, [1.] A. O. OLASUPO, [1.] A. S. AJANI,
[1.] M. A. AYODELE, [1.] A. A. ADEBESIN, [1.] O. S. ODETUNDE

BENJAMIN-BONA-MAHONY EQUATION SOLUTION USING THE LAPLACE HOMOTOPY PERTURBATION METHOD

[1.] Department of Mathematical Sciences, Olabisi Onabanjo University, Ago-Iwoye, NIGERIA

Abstract: The mixture of Laplace transform and Homotopy perturbation was used to solve Benjamin-Bona-Mahony problems. In this research work, three examples of Benjamin-Bona-Mahony equations were considered. The numerical solutions were obtained by Laplace Homotopy Perturbation method. The method was introduced as essential tool to linearize all the associated nonlinear terms in the equations since Laplace transform method alone cannot handle nonlinear terms. The solutions are series form which quickly converges precisely to their exact value with few iterations. The solution obtained by Laplace Homotopy perturbation method accord well with solutions obtained by using other existing methods. The method of Laplace Homotopy perturbation is very powerful integral transform methods in solving some nonlinear Equations like Benjamin-Bona-Mahony equations.
Keywords: Benjamin-Bona-Mahony equations, Laplace Transform, Laplace Homotopy Perturbation method, Waves

1. INTRODUCTION

Nonlinear differential equations played an important role in engineering and sciences. Since most nonlinear problems lack an exact solution, several numerical techniques have been devised to address the issue of nonlinearities. A well-known class of nonlinear partial differential equation which is Benjamin-Bona Mahony (BBM) equation. It may be expressed as [8]

$$u_t(y,t) = u_{yyt}(y,t) - u(y,t)u_y(y,t) - u_y(y,t), \quad u(y,0) = g(y) \tag{1}$$

where uu_y and u_{yyt} are the Dissipative term.

A class of nonlinear partial differential equation which Benjamin-Bona Mahony (BBM) equation which, over time, has undergone extensive research. This equation, was proposed in 1972 to represent the long waves propagating undirectionally in shallow water by Jack Benjamin, Tom Bona, and John Mahony. The equation arises during investigation of water waves and is commonly used to model shallow water wave propagation [2]. The modulation of traveling waves is described by a two-dimensional dispersive wave equation in fluids. In an earlier period, precisely 1966, Peregrine proposed the following equation in his research.

The concept of undular bores is specified in [10], while a more comprehensive n-dimensional rendition of this concept is presented in [14]. Several non-linear evolution equations are significant in the analysis of certain occurrences involving within plasmas, ion acoustic waves exist. Individual structures in dusty plasmas that are affected by magnetic fields are referred to as dust acoustic solitary structures. Electromagnetic radiation present in films of specific sizes is a quantized phenomenon [4]. Various methods were developed to provide solution to the traveling wave problems. Several techniques include: lie group method, inverse scattering technique, Backlund transformation, homogeneous balance technique, Natural Transformation, factorization techniques, Laplace Transform Adomian decomposition techniques, the Pseudo-spectral techniquess, exp-function method, Riccati equation expansion method [12], [6], [9], [15], [3], [5].

This BBM equation has been broadly studied due to its extremely interesting dynamical behavior, which include the formation of solitary waves and the phenomenon of solutions that explode up. The equation for Benjamin-Bona Mahony (BBM), which is majorly called regularized long-wave equation, which is very important for understanding shallow waves as well as drift, wave in plasma, and Rossby waves in fluid rotation. As an example,[12]. It is an upgrade version of the KDV equation, specially outlined for propagating long, moderately amplitude gravity waves that propagate unidirectionally in 1+1 dimensions. It was illustrated that result of the Benjamin-Bona Mahony (BBM) equations are uniquely stable. Korteweg de Vries equation has a limitless line integrals of motion, while the equation of Benjamin-Bona Mahony (BBM) is limited to tripple integrals [7].

The equation of Benjamin-Bona Mahony was initially proposed by Peregrine when when studying undular bores in 1966 [10]. An n-dimensional version was given in a generalized form [14]. Several equations of nonlinear evolution make significant contributions in the investigation of some occurrences as well as dust

ISSN 1584 – 2665 (printed version); ISSN 2601 – 2332 (online); ISSN-L 1584 – 2665

acoustic isolated characteristics in magnetized dusty plasmas, ion acoustic waves, and quantized sized films of electromagnetic waves are all examples of plasmas. [4]. The Riccati equation expansion, the lie group method, the inverse scattering technique, the Natural transform, the homogeneous balance technique, the Bäcklund transformation, the Laplace Adomian decomposition technique, the pseudospectral approach, and the lie group technique were some of the techniques proposed to solve these nonlinear evolution equations with traveling wave solutions ([12], [6], [9], [15], [5], [3]). The aforementioned techniques obtained several forms of nonlinear differential equation solutions.

Several physical domains are familiar with Benjamin-Bona Mahony (BBM) equations [2]. It provides a framework for understanding the performance of long waves by taking the cognizance of both nonlinear and dissipative effects. Acoustic waves in harmonic crystals, Magnetohydrodynamic (MHD) waves in cold plasma, sound-gravity waves in compressible fluids and long wavelength, ripple waves in liquids are a few examples of surface waves, all included the use of this equation [7]. The BBM equation's dynamics have attracted significant attention from mathematicians [12]. The Korteweg de Vries equation’s regular form that was created especially for examining waves in shallow water.

The equation has technological significant benefits compared with the Korteweg de Vries based on the existence and stability properties in several theoretical studies where long waves is being modelled from BBM. The equation gives a model of transmitted waves in one dimension under specified conditions and may be employed to study shallow gravity water waves, Rossby waves in spinning fluids. By considering the associated transformed equations' dispersion relation, it is easiest to identify the most significant difference among the KdV and BBM models. It is clear that these interactions produce radically different reactions to short waves and are only comparable for relatively tiny wave numbers. This is the primary factor why theory of the Benjamin-Bona Mahony equation, which forgot to put into consideration the dissipation and non-integrability, [2], [12].

The equation of Korteweg de Vries model was employed to show extended, low-amplitude nonlinear waves on the surface of a perfect, inviscid fluid. [11] . It is an integrable equation, which its solution can be obtained using the inverse scattering transform. Solitons, which are localized waves with either infinite support or exponential decay, present in the KdV equation because of the delicate balance that weak nonlinearity and dispersion effects maintain. The solutions to the equation of Benjamin-Bona Mahony and the equation of Korteweg de Vries have drawn a lot of interest. Scientist such as Zabusky and Kruskal have examined the interplay between solitary waves with the recurrence of starting states. [1]. “Soliton” was used to describe the fragmental characteristics exhibited by water waves during interactions.

The interaction of two solitons has played a major part in demonstrating the preservation of their shapes, velocity, and the constant pulse-like nature of water waves (Friedman, Partial Differential equations, 1969). To acknowledge the importance of this occurrence using Laplace Homotopy Perturbation Method (LHPM) we investigate the BBM equations analytically. Through this method, Bona-Mahony (BBM) solution is obtained in infinite forms, which approximates to its exact. This allows us to achieve a valuable insight into the characteristics and features of solitons in the context of BBM equations.

2. BASIC PROPERTIES

Here, the basic definitions and properties were given which will be used in this paper.

Definition 2.1 [8]: If $\mathrm{R(t)}$ is a real-valued function that is continuously specified for all $\mathrm{t}, 0 < \mathrm{t} < \infty$ then the Laplace transform of $\mathrm{Q(t)}$ is written as:

$$\mathrm{L[Q(t)] = Q(s) = \int_0^\infty e^{-st} Q(t) dt} \tag{2}$$

in which s is a transform parameter, $\mathrm{S} > 0$ and L is the Laplace operator from $\mathrm{Q(t)}$ into $\mathrm{Q(s)}$.

Theorem 2.2: Let $\mathrm{a_1}$ and $\mathrm{a_2}$ be constants where $\mathrm{Q_1(t)}$ and $\mathrm{Q_2(t)}$are the Laplace transform of $\mathrm{q_1(s)}$and $\mathrm{q_2(s)}$respectively the

$$\mathrm{L\{a_1Q_1(t) + a_2Q_2(t)\} = a_1L\{Q(t)\} + a_2L\{Q_2(t)\}}$$
$$\mathrm{L\{a_1Q_1(t) + a_2Q_2(t)\} = a_1q_1(s) + a_2q_2(s)} \tag{3}$$

Theorem 2.3 (Inverse property): Let $\mathrm{a_1}$ and $\mathrm{a_2}$ be constants where $\mathrm{q_1(s)}$ and $\mathrm{q(s)}$are the Laplace transformation of $\mathrm{Q_1(t)}$and $\mathrm{Q_2(t)}$respectively then

$$\mathrm{L^{-1}\{a_1q_1(s) + a_2q_2(s)\} = a_1L^{-1}\{q(s)\} + a_2L^{-1}\{q_2(s)\}}$$
$$\mathrm{L^{-1}\{a_1q_1(s) + a_2q_2(s)\} = a_1Q_1(t) + a_2Q_2(t)} \tag{4}$$

3. METHOD OF SOLUTION

Considering the Benjamin-Bona-Mahony problem (BBM) of the form [8]:

ISSN 1584 – 2665 (printed version); ISSN 2601 – 2332 (online); ISSN-L 1584 – 2665

$$u_t(y,t) = u_{yyt}(y,t) - u(y,t)u_y(y,t) - u_y(y,t) \tag{5}$$

Subject to initial condition

$$u(y,0) = g(y) \tag{6}$$

Taking the Laplace Transform of equation (5)

$$L\{u_t(y,t)\} = L\{u_{yyt}(y,t)\} - u(y,t)u_y(y,t) - u_y(y,t)\} \tag{7}$$

Using the linearity and differential property of Laplace Transform

$$su(y,s) - u(y,0) = L\{u_{yyt}(y,t)\} - u(y,t)u_y(y,t) - u_y(y,t)\} \tag{8}$$

$$su(y,s) = u(y,0) + L\{u_{yyt}(y,t)\} - u(y,t)u_y(y,t) - u_y(y,t)\} \tag{9}$$

Substituting the initial condition in equation (6) into equation (9) yields

$$su(y,s) = g(y) + L\{u_{yyt}(y,t)\} - u(y,t)u_y(y,t) - u_y(y,t)\} \tag{10}$$

$$u(y,s) = \frac{g(y)}{s} + \frac{1}{s}L\{u_{yyt}(y,t)\} - u(y,t)u_y(y,t) - u_y(y,t)\} \tag{11}$$

Taking inverse Laplace Transform of equation (11) leads to

$$u(y,t) = \frac{g(y)}{s} + L^{-1}\left\{\frac{1}{s}L\{u_{yyt}(y,t)\} - u(y,t)u_y(y,t) - u_y(y,t)\right\} \tag{12}$$

Taking the solution of equation (5) to be of the form

$$u = u_0 + \rho u_1 + \rho^2 u_2 + \rho^3 u_3 + \cdots \tag{13}$$

To consider the nonlinear operator $u(y,t)u_y(y,t)$, here, the Homotopy perturbation approach is used

$$u(y,t) = \sum_{n=0}^{\infty} \rho^n u_n(y,t) \tag{14}$$

Substituting equation (14) into equation (12) to obtain

$$\sum_{n=0}^{\infty} \rho^n u_n(y,t) = \frac{g(y,t)}{s} + L^{-1}\{\frac{1}{s}L\left\{\sum_{n=0}^{\infty} \rho^n u_{n_{yyt}}(y,t) - \left(\sum_{n=0}^{\infty} \rho^n u_n(y,t)\right)\left(\sum_{n=0}^{\infty} \rho^n u_{ny}(y,t)\right) - \sum_{n=0}^{\infty} \rho^n u_{ny}(y,t)\right\} \tag{15}$$

When we compare the coefficients of the corresponding powers of ρ, we get

$$\rho^0:\ u_0(y,t) = \frac{g}{s}(u_n,t)$$

$$\rho^1:\ u_1(y,t) = L^{-1}\left\{\frac{1}{s}L\{u_{0yyt}(y,t) - u_0u_{0y} - u_{0y}\}\right\}$$

$$\rho^2:\ u_2(y,t) = L^{-1}\left\{\frac{1}{s}L\{u_{1yyt}(y,t) - u_1u_{1y} - u_{1y}\}\right\}$$

$$\cdots$$

$$\rho^n:\ u_n(y,t) = L^{-1}\left\{\frac{1}{s}L\{u_{n-1yyt}(y,t) - u_{n-1}u_{(n-1)y} - u_{(n-1)y}\}\right\}$$

The solution can be expressed as

$$u(y,t) = u_0(y,\quad t) + u_1(y,\quad t) + u_2(y,t) + \cdots$$

4. NUMERICAL EXAMPLES

— Example 1

Considering the Benjamin-Bona-Mahony problem of the form [8]:

$$u_t + u_y + uu_y - u_{yyt} = 0 \tag{16}$$

With initial conditions given by

$$u(y,0) = e^y \tag{17}$$

Equations (16) can be written as

$$u_t = -u_y - uu_y + u_{yyt} \tag{18}$$

Taking Laplace Transform of equation (18)

$$L\{u_t\} = L\{-u_y - uu_y + u_{yyt}\} \tag{19}$$

$$su(y,s) - u(y,0) = L\{-u_y - uu_y + u_{yyt}\} \tag{20}$$

Substituting the given initial condition

$$su(y,s) = e^y + L\{-u_y - uu_y + u_{yyt}\} \tag{21}$$

Taking the Laplace Inverse of equation (21) leads to

$$u(y,t) = L^{-1}\left\{\frac{e^y}{s}\right\} + L^{-1}\left\{\frac{1}{s}L\{-u_y - uu_y + u_{yyt}\}\right\} \tag{22}$$

$$u(y,t) = e^y + L^{-1}\left\{\frac{1}{s}L\{-u_y - uu_y + u_{yyt}\}\right\} \tag{23}$$

The homotopy perturbation method can be written as

$$u(y,t) = \sum_{n=0}^{\infty} \rho^n u_n(y,t) \tag{24}$$

ISSN 1584 – 2665 (printed version); ISSN 2601 – 2332 (online); ISSN-L 1584 – 2665

Substituting equation (24) into (23)

$$\sum_{n=0}^{\infty}\rho^n u_n(y,t) = e^y + L^{-1}\left\{\frac{L}{s}\left\{-\sum_{n=0}^{\infty}\rho^n u_n(y,t) - \sum_{n=0}^{\infty}\rho^n u_n(y,t)u_{ny}(y,t) + \sum_{n=0}^{\infty}\rho^n u_{nyyt}(y,t)\right\}\right\} \quad (25)$$

Equating coefficients of the corresponding powers of ρ yield

$$\rho^0:\ \ u_0(y,t) = e^y$$

$$\rho^1:\ \ u_1(y,t) = L^{-1}\left\{\frac{L}{s}\{-u_{0y} - u_0 u_{0y} + u_{0yyt}\}\right\}$$

$$= L^{-1}\left\{\frac{L}{s}\{-e^y - e^y e^y + 0\}\right\} = L^{-1}\left\{\frac{L}{s}\{-e^y - e^{2y}\}\right\} = L^{-1}\left\{-\frac{e^y}{s^2} - \frac{e^{2y}}{s^2}\right\} = -e^y(e^y+1)t$$

$$\rho^2:\ \ u_2(y,t) = L^{-1}\left\{\frac{L}{s}\{-u_{1y} - u_1 u_{0y} - u_0 u_{1y} + u_{1yyt}\}\right\}$$

$$u_2(y,t) = L^{-1}\left\{\frac{L}{s}\{u_{1yyt} - u_1 u_{0y} - u_0 u_{1y} - u_{1y}\}\right\}$$

$$= L^{-1}\left\{\frac{L}{s}\{-4e^{2y} - e^y - (-te^{2y} - te^y)e^y - e^y(-2te^{2y} - te^y) - (-2te^{2y} - te^y)\}\right\}$$

$$u_2(y,t) = L^{-1}\left\{\frac{-e^y(4e^y+1)}{s^2} + \frac{3e^{3y} + 4e^{2y} + e^y}{s^3}\right\}$$

$$= -e^y(4e^y+1) + \frac{t^2}{2!}(3e^{3y} + 4e^{2y} + e^y)$$

Other terms of $u(y,t)$ can be completed by following the same procedure.
Thus, the solution $u(y,t)$ is expressed in the form

$$u(y,t) = u_0(y,t) + u_1(y,t) + u_2(y,t) + \cdots$$

$$u(y,t) = e^y - e^y(4e^y+1)t - e^y(4e^y+1)t + \frac{t^2}{2!}(3e^{3y} + 4e^{2y} + e^y) \quad (26)$$

The equation (26) which is the solution of equation (16) agrees with the solution obtained in [8]

— Example 2

Considering the Benjamin-Bona-Mahony problem of the form [2]

$$u_t(y,t) = u_{yyt}(y,t) - uu_y(y,t) - u_y(y,t) \quad (27)$$

With initial condition given by

$$u(y,0) = \operatorname{sech}^2\left(\frac{y}{4}\right) \quad (28)$$

Taking the Laplace Transform of equation (27)

$$L\{u_t(y,t)\} = L\{u_{yyt}(y,t) - uu_y(y,t) - u_y(y,t)\} \quad (29)$$

Applying the differential property of the Laplace Transform

$$su(y,s) - u(y,0) = L\{u_{yyt}(y,t) - uu_y(y,t) - u_y(y,t)\} \quad (30)$$

Substituting the initial condition in equation (28) in equation (30)

$$su(y,s) - \operatorname{sech}^2\left(\frac{y}{4}\right) = L\{u_{yyt} - uu_y - u_y\} \quad (31)$$

$$u(y,s) = \frac{1}{s}\operatorname{sech}^2\frac{y}{4} + \frac{1}{s}L\{u_{yyt} - uu_y - u_y\} \quad (32)$$

Taking the Laplace Inverse of equation (32) we obtain

$$u(y,t) = L^{-1}\left\{\frac{1}{s}\operatorname{sech}^2\frac{y}{4}\right\} + L^{-1}\left\{\frac{1}{s}L\{u_{yyt} - uu_y - u_y\}\right\} \quad (33)$$

$$u(y,t) = \operatorname{sech}^2\frac{y}{4} + L^{-1}\left\{\frac{1}{s}L\{u_{yyt} - uu_y - u_y\}\right\} \quad (34)$$

Applying the homotopy perturbation method to take care of the nonlinearities in equation (34)
Recall in equation (14) that

$$u(x,t) = \sum_{n=0}^{\infty}\rho^n u_n(y,t)$$

Substituting equation (14) into (34)

$$\sum_{n=0}^{\infty}\rho^n u_n(y,t) = \operatorname{sech}^2\frac{y}{4} + L^{-1}\left\{\frac{L}{s}\left\{\sum_{n=0}^{\infty}\rho^n u_{nyyt}(y,t) - \sum_{n=0}^{\infty}\rho^n u_{ny}(y,t) - \sum_{n=0}^{\infty}\rho^n u_{ny}(y,t)\right\}\right\} \quad (35)$$

Equating coefficients of the corresponding powers of ρ

$$\rho^0\ \ :\ \ u_0(x,t) = \operatorname{sech}^2\frac{y}{4}$$

ISSN 1584 – 2665 (printed version); ISSN 2601 – 2332 (online); ISSN-L 1584 – 2665

$$\rho^1 \quad : \quad u_1(y,t) = L^{-1}\left\{\frac{L}{s}\{u_{0yyt} - u_0u_{0y} + u_{0y}\}\right\}$$

$$u_1(y,t) = [\frac{1}{2}\mathrm{sech}^4\left(\frac{y}{4}\right)\tanh\left(\frac{y}{4}\right) + \frac{1}{2}\mathrm{sech}^2\left(\frac{y}{4}\right)\tanh\left(\frac{y}{4}\right)$$

$$\rho^2 \quad : \quad u_2(y,t) = L^{-1}\left\{\frac{L}{s}[u_{1yyt}] - [u_1u_{0y} + u_0u_{1y}] - [u_{0y}]\right\}$$

$$= \frac{1}{256} + \mathrm{sech}^8\left(-104t + 23t\cosh\left(\frac{y}{2}\right)\right) + [\frac{1}{256} + \mathrm{sech}^8(16 + \cosh y + t\cosh\left(\frac{3y}{2}\right)$$

$$+ \left[\frac{1}{256} + \mathrm{sech}^8\left(-107\sinh\frac{y}{2} + 8\sinh y + \sinh\frac{3y}{2}\right)\right]$$

Thus, the result $u(y,t)$ is expressed in the form

$$u(y,t) = u_0(y,t) + u_1(y,t) + u_2(y,t) + \cdots \tag{36}$$

$$u(y,t) = \mathrm{sech}^2\left(\frac{y}{4}\right) + \left[\frac{1}{2}\mathrm{sech}^8\left(\frac{y}{4}\right)\tanh\left(\frac{y}{4}\right) + \frac{1}{2}\mathrm{sech}^2\left(\frac{y}{4}\right)\tanh\left(\frac{y}{4}\right)\right]t$$
$$+ \left[\frac{1}{256} + \mathrm{sech}^8\left(-104t + 23t\cosh\left(\frac{y}{2}\right)\right)\right] + \left[\frac{1}{256} + \mathrm{sech}^8\left(16t\cosh(y) + t\cosh\left(\frac{3y}{2}\right)\right)\right]$$
$$+ \left[\frac{1}{256} + \mathrm{sech}^8\left(-107\sinh\left(\frac{y}{2}\right) + 8\sinh + \sinh\left(\frac{3y}{2}\right)\right)\right] \tag{37}$$

Equation (37) also has a closed form that reads as

$$u(y,t) = \mathrm{sech}^2\left(\frac{y}{4} - \frac{t}{3}\right) \tag{38}$$

The equation (38) which is the solution of equation (27) agrees with the solution obtained in [2].

— Example 3

Consider the Benjamin-Bona-Mahony problem of the form [8]

$$u_t(y,t) = u_{yyt}(y,t) - uu_y(y,t) - u_x(y,t) \tag{39}$$

Subject to initial condition given by

$$u(x,0) = y^2 \tag{40}$$

Taking the Laplace Transform of equation (39)

$$L\{u_t\} = L\{u_{yyt} - uu_y - u_y\} \tag{41}$$

$$su(y,s) - u(y,0) = L\{u_{yyt} - uu_y - u_y\} \tag{42}$$

Substituting the given initial condition

$$su(y,s) = y^2 + L\{u_{yyt} - uu_y - u_y\} \tag{43}$$

$$u(y,s) = \frac{y^2}{s} + \frac{1}{s}L\{u_{yyt} - uu_y - u_y\} \tag{44}$$

Taking the Inverse Laplace Transform of equation (42) leads to

$$u(y,t) = L + \left\{\frac{y^2}{3}\right\} + L^{-1}\left\{\frac{1}{s}L\{u_{yyt} - uu_y - u_y\}\right\} \tag{45}$$

$$u(y,t) = y^2 + L^{-1}\left\{\frac{1}{s}L\{u_{yyt} - uu_y - u_y\}\right\} \tag{46}$$

Recall in equation (14) that the homotopy perturbation method can be written as

$$u(y,t) = \sum_{n=0}^{\infty}\rho^n u_n(y,t)$$

Substituting equation (4) into equation (46) yields

$$\sum_{n=0}^{\infty}\rho^n u_n(y,t) = y^2 + L^{-1}\left\{\frac{1}{s}\left\{\sum_{n=0}^{\infty}\rho^n u_{nyyt}(y,t) - \sum_{n=0}^{\infty}\rho^n u_n(y,t)\,u_{ny}(y,t) - \sum_{n=0}^{\infty}\rho^n u_{ny}(y,t)\right\}\right\} \tag{47}$$

Equating coefficients of the corresponding powers of ρ, yields

$$\rho^0 \quad : \quad u_0(y,t) = y^2$$

$$\rho^1 \quad : \quad u_1(y,t) = L^{-1}\left\{\frac{L}{s}\left\{[u_{0yyt}] - [u_0u_{0y}] - [u_{0y}]\right\}\right\}$$

$$u_1(y,t) = L^{-1}\left\{\frac{L}{s}\{[0] - (y^2)(2y) - 2y\}\right\}$$

$$= L^{-1}\left\{\frac{L}{s}\{-2y^3 - 2y\}\right\} = L^{-1}\left\{\frac{L}{s}\left[\frac{-2y^3 - 2y}{s}\right]\right\} = L^{-1}\left\{\frac{-2y^3 - 2y}{s}\right\} = -2y(y^2 + 1)t$$

ISSN 1584 – 2665 (printed version); ISSN 2601 – 2332 (online); ISSN-L 1584 – 2665

$$\rho^2 \quad : \quad u_2(y,t) = L^{-1}\left\{\frac{L}{s}\left\{[u_{1yyt}] - [u_1 u_{0y} + u_0 u_{1y}] - [u_{1y}]\right\}\right\}$$

$$u_2(y,t) = L^{-1}\left\{\frac{L}{s}\{[-12y] - [(-2y^3t - 2yt)(2y)] + [y^2(-6y^2t - 2t)] - [-6y^2t - 2t]\ \}\right\}$$

$$= L^{-1}\left\{\frac{L}{s}\left[\frac{-12y}{s} + \frac{4y^4}{s^2} + \frac{4y^2}{s^2} + \frac{4y^2}{s^2} + \frac{6y^4}{s^2} + \frac{2y^2}{s^2} + \frac{6y^2}{s^2} + \frac{2}{s^2}\right]\right\}$$

$$= L^{-1}\left\{\frac{12y}{s^2} + \frac{10y^4 + 12y^2 + 2}{s^3}\right\}$$

$$u_2(y,t) = 12yt + (5y^4 + 6y^2 + 1)t^2$$

Other terns can be obtained by following the same procedure

Thus, the result $u(y,t)$ can be expressed as

$$u(y,t) = u_0(y,t) + u_1(y,t) + u_2(y,t) + \cdots$$

$$u(x,t) = y^2 - 2y^3(y^2+1)t - 12yt + (5y^4 + 6y^2 + 1)t^2 \tag{48}$$

$$= y^2 - (2y^3 + 14y)t + (5y^4 + 6y^2 + 1)t^2 \tag{49}$$

The equation (49) which is the solution of equation (39) agrees perfectly with the solution obtained in [8].

4. CONCLUSION

Here, LHPM has been used to obtain the approximate results for the Benjamin-Bona-Mahony problems. This method's main benefit is that it provides an analytical approximate solution in series of sequence which converges rapidly. The results obtained show that Laplace Homotopy Perturbation Method demonstrates its reliability and signifies a significant improvement in tackling nonlinear partial differential equations over other established techniques.

References

[1] Adoms, R. A. (1975). Sobolev Spaces. New York: Academic Press.
[2] Benjamin, T.B., Bona, J.L., Mahony, J.J. (1972). Model equations for long waves in non;linear dispersive systems. Philosophical Transactions of the Royal Society, A(272), 47-78.
[3] Calvert, B. (1976). The equation A (r, u(t)) + E (t, u(t)) = 0. Mathematical Proceedings of the Cambridge Philosophical Society, 79, 545–561.
[4] Friedman, A. (1969). Partial Differential equations,. New York.: Holt, Rinehart & Winston.
[5] Haq, F., Shah, K., Khan, A., Shahzad, M., Rahman, G. (2017). Numerical solution of fractional order epidemic model of a vector born disease by Laplace Adomian decomposition method. Punjab University Journal of Mathematics, 49(2), 13-22.
[6] Johnpillai, A. G., Khalique, C. M. (2011). Variational approaches to conservation laws for a nonlinear evolution equation with time dependent coefficients. Quaestiones Mathematicale, 34, 235-245.
[7] Molati, M., Khalique, C., M. (2012). Lie symmetry analysis of the timevariable. Advances in Difference Equations, 212.
[8] Muhammmad I., Abbas M, Atiq U. R. (2019). Analytic Solution to Benjamin-Bona- Mahony Equation by Using Laplace Adomian Decomposition Method. Matrix Science Mathematic, 3(1), 01-04.
[9] Olver, P. J. (1993). Applications of lie groups to differential equations. NY, USA.: Springer-Verlag: New York,.
[10] Peregrine, D. H. (1966). Calculations of the development of an undularbore. Journal of Fluid Mechanics, 25, 321.
[11] Shah, K., Khali, H., Kha, R. A. (2018). Analytical Solutions of Fractional Order Diffusion Equations by Natural Transform Method. Iranian Journal of Science and Technology, Transactions A: Science, 42(3), 1479-1490.
[12] Singh, k., Gupta, R., K. (2006). Lie symmetries and exact solutions of a new generalized Hirotasatsuma coupled KDV system with variable coefficients. International Journal of Engineering Science, 44, 241-255.
[13] Singh, K., Gupta, R., K., Kumar, S. (2011). Benjamin-Bona- Mahony (BBM) equation with variable coefficients similarity reductions and Painleve analysis. Applied Mathematics and Computation, 217, 7021-7027.
[14] Wang, M. L., Wang, Y. M. (2001). A new Backlund transformation and multi-solution, solutions to the KDV equation with general variable coefficient. Physics Letters, A. 287, 211-216.
[15] Witham, G. G. (1974). Linear and Nonlinear Waves. New York: John Wiley.

ISSN 1584 – 2665 (printed version); ISSN 2601 – 2332 (online); ISSN–L 1584 – 2665