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NUMERICAL APPROXIMATION OF NON LINEAR DELAY DIFFERENTIAL EQUATION USING SAWI ITERATIVE METHOD

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Abstract: The Sawi Transform Iterative Method (STIM), an effective analytical technique for olving nonlinear delay differential equations (NDDE), is presented in this work. Here, this approach combines the novel iterative method with the Sawi transform method for nonlinear equations. This approach solves the equations immediately and consistently without requiring a lot of computer work, linearization, or perturbations. The efficiency and dependability of the approach are validated by solving three example DDE scenarios. After adding up all eight approximate solution's iterations, the outcomes are compared with the exact solutions via tables and graphs. Based on the results obtained it was suggested that other classes of nonlinear differential equations may be solved using this method.
Keywords: Sawi Transform, Differential Equation, Integral Transform, Nonlinear Equation, Linear Equation

1. INTRODUCTION

Mathematical models called delay differential equations (DDEs) are employed to illustrate problems in physics, engineering, biomathematics, and other fields. At two different time instants (past and present), the derivatives of unknown functions are involved in these equations. Mathematical models including DDEs are often encountered by researchers in the engineering and biological sciences fields (Singh, 2021; Perviaz et al, 2021; Nakata, 2022; Mahatekar et al., 2021). Functional differential equations (FDEs) indicate a more versatile version of differential equations. The most basically and naturally occurring form of FDEs is the delay differential equation. Dynamical system classes, or DDEs, are extensively employed in a wide range of disciplines, including mechanics, economics, biology, ecology, physiology, and epidemiology. Many dynamic processes include time delays, which need to be taken into account in any correct model of these processes (Jumaa, 2017).

In order to get approximate or precise solutions for both DDEs and nonlinear DEs, several researchers have recently devised and examined a variety of analytical and numerical approaches (Ali 2022, Deresse et al., 2021 & Deresse, 2022). The authors of the paper (Vilu et al., 2019, Deresse, 2022, Malikov et al., 2020, Srivastava, 2020 & Amad et al., 2020) used the variational iteration technique (VIM) to find a preliminary solution to nonlinear DDEs. (Ghaffara et al., 2022) integrated the Padè approximation, Tarig transformation method, and differential transform approach to solve delayed protein degradation and delayed vector-borne illness models. By using this approach, the approximate solutions' region of convergence is expanded via the application of Padè approximation. Several NDDEs' numerical findings were generated with the assistance of Kumar and Methi (2021). This study explores the application of Sawi Transform Iteration Method (STIM) to Nonlinear Delay Differential Equation (NDDE).

2. MATERIALS AND METHODS

Delay Differential Equation

A differential equation in which a time delay is incorporated and the derivative of a function at a given time relies on both its value at that time and its values at earlier periods is known as a delay differential equation.

$$f^{(n)} = f\big(t, y(t), f(t-\tau_1), \dots, f(t-\tau_i)\big), \ t \ge 0 \tag{1}$$
$$f(t) = g(t)$$

Here, $g(t)$ is the initial function, $\tau_i, \ i > 0$ is called the delay or lag function, f is given function with $\tau_i(t) \le t$. If $\tau_i > 0$ is a constant, it is a constant dependent delay; If $\tau_i(t) \ge 0$ is time dependent, it is time dependent and it $u\big(\tau_i(t)\big) \ge 0$ is state dependent delay.

Sawi Transform

The Sawi transform, originating from the classical Fourier integral, is a mathematically simple method used to solve differential equations (Sawi, T.M. and Ezaki, S.M. 2011). Sawi transform is a new transform designed for functions of exponential order, focusing on functions defined by:

$$A = \left\{R(t): \ \exists, M, l_1, l_2 > 0. \ |R(t)| < Me^{\left(\frac{|t|}{k_j}\right)}, \ \text{If } t \in (-1)^j \times [0, \infty)\right\} \tag{2}$$

The integral equations for a function in set A define the Sawi transform, represented by the operator S, which needs a finite constant M.

$$S\{R(t)\} = T(v) = \frac{1}{v^2}\int_0^\infty R(t)\ e^{-\frac{t}{v}}dt.\quad t \geq 0,\quad l_1 \leq v \leq l_2 \tag{3}$$

Properties of Sawi Transform

— Linearity Property of Sawi Transform:

Given the Sawi Transform of functions $R(t)$ and $W(t)$ are $R(z,v)$ and $W(z,v)$ respectively, then Sawi transform of $[pR(z,t) + qW(z,t)]$ can be given as $[pR(z,v) + qW(z,v)]$ where p and q are arbitrary constants.

Proof: Let $R(z,t)$ and $W(z,t)$ be any two functions whose Sawi transform with respect to exist. For random constants p and q, we have

$$S\{pR(z,t) + qW(z,t)\} = pS\{R(z,t)\} + qS\{W(z,t)\} \tag{4}$$

Based on the definition of Sawi transform, yields

$$S\{pR(z,t) + qW(z,t)\} = pS\{R(z,t)\} + qS\{W(z,t) \tag{5}$$

$$S\{pR(z,t) + qW(z,t)\} = \frac{1}{v}\int_0^\infty \big(pR(z,t) + qW(z,t)\big)e^{-vt}\,dt \tag{6}$$

$$= p\left(\frac{1}{v}\int_0^\infty \big(R(z,t)\big)e^{(-vt)dt}\right) + q\left(\frac{1}{v}\int_0^\infty W(z,t)e^{-vt}\,dt\right) \tag{7}$$

$$= pS\{R(z,t)\} + qS\{W(z,t)\} \tag{8}$$

— Derivative Property of Sawi Transform

If $S\{R(t)\} = T(v)$, Then,

$$S\{R'(t)\} = \frac{1}{v}T(v) - \frac{1}{v^2}R(0) \tag{9}$$

$$S\{R''(t)\} = \frac{1}{v^2}T(v) - \frac{1}{v^3}R(0) - \frac{1}{v^2}R'(0) \tag{10}$$

$$S\{R^{(n)}\ (t)\} = \frac{1}{v^2}T(v) - \sum_{k=0}^{n-1}\left(\frac{1}{v}\right)^{n-(k-1)} R^{(k)}(0) \tag{11}$$

Table 1. Fundamental Properties of Sawi Transform (Higazy, & Aggarwal, 2021).

S/N	Property	Mathematical Form
1.	Linearity	$S\{pG_1(z,t) + qG_2(z,t)\} = pS\{G_1(z,t)\} + qS\{G_2(z,t)\}$
2.	Change of Scale	$S\{G(pt)\} = pg(pv)$
3.	Shifting	$S\{e^{pt}G(t)\} = \left(\frac{1}{1-pv}\right)^2 g\left(\frac{v}{1-pv}\right)^v$
	First Derivative	$S\{g'(t)\} = \frac{1}{v}g(v) - \frac{1}{v^2}g(0)$
5.	Second Derivative	$S\{g''(t)\} = \frac{1}{v^2}g(v) - \frac{1}{v^3}g(0) - \frac{1}{v^2}g'(0)$
6.	nth Derivative	$S\{g^{(n)}(t)\} = \frac{1}{v^n}g(v) - \frac{1}{v^{n+1}}g(0) - \frac{1}{v^n}g'(0) \ldots - \frac{1}{v^2}g\text{^}(n-1)(0)$
7.	Convolution	$S\{W_1(t) * W_2(t)\} = v^2S\{W_1(t)\}S\{W_2(t)\}$

Table 2: Sawi transform of some frequently encountered functions (Higazy, & Aggarwal, 2021).

S/N	$R(t)$	$S\{R(t)\} = r(v)$
1.	1	$\frac{1}{v}$
2.	t	1
3.	t^2	$2!\,v$
4.	$t^n,\quad n \in N$	$n!\,v^{n-1}$
5.	$t^n,\qquad n > -1$	$\Gamma(n+1)v^{n-1}$
6.	e^{pt}	$\frac{1}{v(1-pv)}$
7.	$\sin pt$	$\frac{p}{1+p^2v^2}$
8.	$\cos pt$	$\frac{1}{v(1+p^2v^2)}$
9.	$\sinh pt$	$\frac{p}{1-p^2v^2}$
10.	$\cosh pt$	$\frac{1}{v(1-p^2v^2)}$

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Table 3: Inverse Sawi Transform (Higazy, & Aggarwal, 2021).

S/N	$w(v)$	$W(t) = S^{-1}\{w(v)\}$
1.	$\frac{1}{v}$	1
2.	1	t
3.	v	$\frac{t^2}{2!}$
4.	$v^{n-1} \quad n \in N$	$\frac{t^n}{n!}$
5.	$v^{n-1} \quad n > -1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{1}{v(1-pv)}$	e^{pt}
7.	$\frac{1}{1+(pv)^2}$	$\frac{\sin pt}{p}$
8.	$\frac{1}{v(1+(pv)^2)}$	$\cos pt$
9.	$\frac{1}{1-(pv)^2}$	$\frac{\sinh pt}{p}$
10.	$\frac{1}{v(1-p^2v^2)}$	$\cosh pt$

Sawi Iteration Method (SIM)

Considering the general nonlinear Delay Differential Equation of the form (Tsegaye, Deresse, 2022 & Rezapour et al. 2022):

$$\frac{d^n f(t)}{dt^n} + Rf(t) + Mf(t) = W(t) \quad n = 1,2,3,4 \tag{12}$$

Subject to the initial condition

$$\left.\frac{d^{n-1} f(t)}{dt^n}\right|_{x=0} = w_{n-1}(t), \qquad n = 1,2,3,4, \dots \tag{13}$$

refers to the derivative of $f(t)$ of the order n, $Mf(t)$ refers to the nonlinear term, R is a linear operator and $W(t)$ refers to the source term.

Taking the Sawi Transform of equation (13), we obtain

$$S\left[\frac{d^n f(t)}{dt^n}\right] + S[Rf(t)] + S[Mf(t)] = S[W(t)] \quad n = 1,2,3,4 \tag{14}$$

Invoking the differential property of Sawi Transform, yields

$$\frac{1}{v^n} S[f(t)] - \sum_{k=0}^{n-1} \frac{1}{v^{n-k+1}} \frac{d^n f(0)}{dt^n} + S[Rf(t) + Mf(t)] = S[W(t)] \tag{15}$$

Multiplying the equation by v^n, then we have

$$S[f(t)] = \sum_{k=0}^{n-1} \frac{v^n}{v^{n-k+1}} \frac{d^k f(0)}{dt^k} + v^n S[W(t)] - v^n S[Rf(t) + Mf(t)] \qquad 16)$$

$$S[f(t)] = \sum_{k=0}^{n-1} v^{m-m+k-1} \frac{d^k f(0)}{dt^k} + v^n S[G(t)] - v^n S[Rf(t) + Mf(t)] \tag{17}$$

$$S[f(t)] = \sum_{k=0}^{n-1} v^{k-1} \frac{d^k f(0)}{dt^k} + v^n S[W(t)] - v^n S[Rf(t) + Mf(t)] \tag{18}$$

Taking the Sawi Inverse of equation (18)

$$\sum_{n=0}^{\infty} f_n(t) = S^{-1}\left\{\sum_{k=0}^{n-1} v^{k-1} \frac{d^k f(0)}{dt^k}\right\} + S^{-1}\{v^n S[W(t)]\} - S^{-1}\{v^n S[R \sum_{k=0}^{\infty} A_m(t)]\} \tag{19}$$

where

$$A_m(t) = \sum_{r=0}^{k} f_r(t) f_{k-r}(t) \tag{20}$$

By comparison of equation (19), the components of components of $f_n(t)$ becomes

$$f_0(t) = S^{-1}\left\{\sum_{k=0}^{m-1} v^{k-1} \frac{d^k f(0)}{dt^k}\right\} + S^{-1}\left[v^n S[W(t)]\right] \tag{1}$$

$$f_1(t) = -S^{-1}[vS\{Rf_0(t) + A_0(t)\}]$$

$$f_2(t) = -S^{-1}[vS\{Rf_1(t) + A_1(t)\}]$$

$$\vdots$$

$$f_3(t) = -S^{-1}[vS\{Rf_2(t) + A_2(t)\}]$$

The approximate solution of equation (3.12) is

$$\sum_{0}^{\infty} f(x,t) = f_0(t) + f_1(t) + f_2(t) + f_3(t) \tag{2}$$

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3. RESULT AND DISCUSSION

Example 2.1 Considering the nonlinear Delay Differential Equation of the form (Mohyud-Din & Yildirim, 2010):

$$\frac{d^3f}{dt^3} = -1 + 2f^2\left(\frac{t}{2}\right), \quad 0 \le x \le 1 \tag{23}$$

Subject to the given initial condition

$$f(0) = 1 \qquad f'(0) = 0 \quad f''(0) = 0$$

The exact solution of equation (23) is

$$f(t) = \sin t$$

Taking Sawi Transform of equation (23),

$$S\left[\frac{d^3f}{dt^3} = -1 + 2f^2\left(\frac{t}{2}\right)\right]$$

$$S\left[\frac{d^3f(t)}{dt^3}\right] = -S[1] + 2S\left[f^2\left(\frac{t}{2}\right)\right] \tag{24}$$

By the Sawi differential property we have

$$\frac{1}{v^3}S[f(t)] - \frac{1}{v^4}f(0) - \frac{1}{v^3}f'(0) - \frac{1}{v^2}f''(0) = -\frac{1}{v} + 2S\left[f^2\left(\frac{t}{2}\right)\right] \tag{25}$$

Multiplying equation (25) by v^3, we obtain

$$S[f(t)] = \frac{1}{v}f(0) + f'(0) + vf''(0) - v^2 + 2v^3S\left[f^2\left(\frac{t}{2}\right)\right] \tag{26}$$

Taking the inverse Sawi transform of equation (26), we have

$$f(t) = S^{-1}\left\{\frac{1}{v}f(0) + f'(0) + vf''(0) - v^2 + 2v^3S\left[f^2\left(\frac{t}{2}\right)\right]\right\} \tag{27}$$

Substituting the initial condition into equation (27). Then, we have,

$$f(t) = S^{-1}\{1 - v^2\} + 2S^{-1}\left\{v^3S\left[f^2\left(\frac{t}{2}\right)\right]\right\} \tag{28}$$

According to the Sawi transform table, we have

$$f(t) = t - \frac{t^3}{3!} + 2S^{-1}\left\{v^3S\left[f^2\left(\frac{t}{2}\right)\right]\right\} \tag{29}$$

Therefore, the equation (29) becomes

$$f_0(t) = t - \frac{t^3}{3!} \tag{30}$$

$$f_{n+1}(t) = 2S^{-1}\left[v^3S\left[A_n\left(\frac{t}{2}\right)\right]\right] \qquad n \ge 0 \tag{31}$$

where

$$A_n\left(\frac{t}{2}\right) = \sum_{r=0}^{k} f_r\left(\frac{t}{2}\right) f_{k-r}\left(\frac{t}{2}\right)$$

When $n = 0$, we have

$$A_0\left(\frac{t}{2}\right) = \frac{t^2}{4}$$

Then

$$= 2S^{-1}\left[v^3\,S\left(\frac{t^2}{4}\right)\right] = \frac{1}{2}S^{-1}[v^3 \times 2!\,v] = S^{-1}[v^4] = \frac{t^5}{5!}$$

When $n = 1$, we have

$$A_1\left(\frac{t}{2}\right) = -\frac{t^4}{48}$$

Then

$$f_1(t) = 2S^{-1}\left[v^3S\left(-\frac{t^4}{48}\right)\right] = -\frac{1}{24}S^{-1}[v^3 \times 4!\,v^3] = -S^{-1}[v^6] = -\frac{t^7}{7!}$$

When $n = 2$, we have

$$A_2\left(\frac{t}{2}\right) = \frac{t^6}{1440}$$

Then

$$f_3(t) = 2S^{-1}\left[v^3S\left(\frac{t^6}{1440}\right)\right] = \frac{1}{720}S^{-1}[v^3 \times 6!\,v^5] = S^{-1}[v^8] = \frac{t^9}{9!}$$

When $n = 2$, we have

Therefore, the approximate solution is expressed as

$$f(t) = \sum_{n=0}^{N} f_n(t) = f_0(t) + f_1(t) + f_2(t) + f_3(t) + \cdots \qquad (32)$$

$$= t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \frac{t^9}{9!} - \cdots \qquad (33)$$

Then, the solution can be written in exact form as

$$f(t) = \sin t \qquad (34)$$

Equation (34) which is the resulting exact solution of equation (23) which is the same as the result obtained in (Mohyud-Din & Yildirim, 2010).

- Example 2.2 Considering the nonlinear Delay Differential Equation (NDDE) of the form (Tsegaye, Deresse, 2022 & Rezapour et al. 2022):

$$\frac{df}{dt} = 1 - 2f^2\left(\frac{t}{2}\right), \quad 0 \le x \le 1 \qquad (35)$$

Subject to the initial condition

$$f(0) = 0$$

The exact solution of equation (35) is

$$f(t) = \sin t$$

Taking the Sawi Transform of equation (35), yields

$$S\left[\frac{df(t)}{dt} = 1 - 2f^2\left(\frac{t}{2}\right)\right]$$

Applying the Sawi linearity property

$$S\left[\frac{df(t)}{dt}\right] = S[1] - 2S\left[f^2\left(\frac{t}{2}\right)\right] \qquad (36)$$

By the Sawi differential property we have

$$\frac{1}{v}S[f(t)] = \frac{f(0)}{v^2} + \frac{1}{v} - 2S\left[f^2\left(\frac{t}{2}\right)\right] \qquad (37)$$

Multiplying equation (37) by v, we obtain

$$S[f(t)] = \frac{f(0)}{v} + 1 - 2vS\left[f^2\left(\frac{t}{2}\right)\right] \qquad (38)$$

Taking the inverse Sawi transform of equation (39)

$$f(t) = S^{-1}\left\{\frac{f(0)}{v} + 1 - 2vS\left[f^2\left(\frac{t}{2}\right)\right]\right\} \qquad (39)$$

Substituting $f(0) = 0$ into equation (39). Then, we have,

$$f(t) = S^{-1}\{1\} - 2S^{-1}\left\{vS\left[f^2\left(\frac{t}{2}\right)\right]\right\} \qquad (40)$$

According to the Sawi transform table, we have

$$f(t) = t - 2S^{-1}\left\{vS\left[f^2\left(\frac{t}{2}\right)\right]\right\} \qquad (41)$$

Therefore, the equation (41) becomes

$$f_0(t) = t \qquad (42)$$

$$f_{n+1}(t) = -2S^{-1}\left[vS\left[A_n\left(\frac{t}{2}\right)\right]\right] \quad n \ge 0 \qquad (43)$$

where

$$A_n\left(\frac{t}{2}\right) = \sum_{r=0}^{k} f_r\left(\frac{t}{2}\right) f_{k-r}\left(\frac{t}{2}\right)$$

When $n = 0$, we have

$$A_0\left(\frac{t}{2}\right) = \frac{t^2}{4}$$

Then

$$= -\frac{1}{2}S^{-1}[v \times 2!\, v] = -S^{-1}[v^2] = -\frac{t^3}{3!}$$

When $n = 1$, we have

$$A_1\left(\frac{t}{2}\right) = -\frac{t^4}{48}$$

Then

$$f_1(t) = -2S^{-1}\left[vS\left(-\frac{t^4}{48}\right)\right] = \frac{1}{24}S^{-1}[v \times 4!\, v^3] = S^{-1}[v^4] = \frac{t^5}{5!}$$

When $n = 2$, we have

$$A_2\left(\frac{t}{2}\right) = \frac{t^6}{1440}$$

Then

$$f_3(t) = -2S^{-1}\left[vS\left(\frac{t^6}{1440}\right)\right] = -\frac{1}{720}S^{-1}[v \times 6!\, v^5] = -S^{-1}[v^6] = -\frac{t^7}{7!}$$

When $n = 2$, we have

$$A_3\left(\frac{t}{2}\right) = -\frac{t^8}{80640}$$

Then

$$f_4(t) = -2S^{-1}\left[vS\left(-\frac{t^8}{80640}\right)\right] = \frac{1}{40320}S^{-1}[v \times 8!\, v^7] = S^{-1}[v^8] = \frac{t^9}{9!}$$

Therefore, the approximate solution is expressed as

$$f(t) = \sum_{n=0}^{N} f_n(t) = f_0(t) + f_1(t) + f_2(t) + f_3(t) + \cdots \quad (44)$$

$$= t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \frac{t^9}{9!} + \cdots \quad (45)$$

Then, the solution can be written in exact form as

$$f(t) = \sin t \quad (46)$$

Equation (46) which is the resulting exact solution of equation (35) which is the same as the result obtained in (Tsegaye, Deresse, 2022 & Rezapour et al. 2022)

- Example 2.3 Considering the nonlinear delay differential equation of the form (Mohyud-Din & Yildirim, 2010):

$$\frac{df}{dt} = \frac{1}{2}e^{\frac{t}{2}}f\left(\frac{t}{2}\right) + \frac{1}{2}f(t), \qquad 0 \le x \le 1 \quad (47)$$

Subject to the initial condition

$$f(0) = 1$$

Taking Sawi Transform of equation (25), yields

$$S\left[\frac{df}{dt} = \frac{1}{2}e^{\frac{t}{2}}f\left(\frac{t}{2}\right) + \frac{1}{2}f(t)\right]$$
$$S\left[\frac{df(t)}{dt}\right] = \frac{1}{2}S\left[e^{\frac{t}{2}}f\left(\frac{t}{2}\right) + f(t)\right] \quad (48)$$

Applying Sawi differential property we have

$$\frac{1}{v}S[f(t)] - \frac{f(0)}{v^2} = \frac{1}{2}S\left[e^{\frac{t}{2}}f\left(\frac{t}{2}\right) + f(t)\right] \quad (49)$$

Multiplying equation (27) by v, we obtain

$$S[f(t)] = \frac{f(0)}{v} + \frac{1}{2}vS\left[e^{\frac{t}{2}}f\left(\frac{t}{2}\right) + f(t)\right] \quad (50)$$

Taking the inverse Sawi transform equation of (50), we obtain

$$f(t) = S^{-1}\left\{\frac{f(0)}{v} + \frac{1}{2}vS\left[e^{\frac{t}{2}}f\left(\frac{t}{2}\right) + f(t)\right]\right\} \quad (51)$$

Substituting $f(0) = 1$ into equation (51). Then, we have,

$$f(t) = S^{-1}\left\{\frac{1}{v}\right\} + \frac{1}{2}S^{-1}\left\{vS\left[e^{\frac{t}{2}}f\left(\frac{t}{2}\right) + f(t)\right]\right\} \quad (52)$$

According to the Sawi transform table, we have

$$f(t) = 1 + \frac{1}{2}S^{-1}\left\{vS\left[e^{\frac{t}{2}}f\left(\frac{t}{2}\right) + f(t)\right]\right\} \quad (53)$$

Therefore, the equation (53) becomes

$$f_0(t) = 1 \quad (54)$$

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$$f_{n+1}(t) = \frac{1}{2}S^{-1}\left\{vS\left[e^{\frac{t}{2}}f_n\left(\frac{t}{2}\right) + f_n(t)\right]\right\} \qquad n \geq 0 \tag{55}$$

When $n = 0$, we have

$$f_1(t) = \frac{1}{2}S^{-1}\left\{vS\left[e^{\frac{t}{2}} + 1\right]\right\} \tag{56}$$

Then

$$= \frac{1}{2}S^{-1}\left\{vS\left[e^{\frac{t}{2}}\right] + vS[1]\right\} \tag{57}$$

But

$$S\left[e^{\frac{t}{2}}\right] = \frac{1}{v\left(1 - \frac{1}{2}v\right)} \tag{58}$$

$$v\left[\frac{1}{v\left(1 - \frac{1}{2}v\right)}\right] = \frac{2}{2 - v} = 2\sum_{n=0}^{\infty}\frac{1}{2}\left(\frac{v}{2}\right)^n = \sum_{n=0}^{\infty}\left(\frac{v}{2}\right)^n = 1 + \frac{v}{2} + \frac{v^2}{4} + \frac{v^3}{8} + \frac{v^4}{16} + \cdots \tag{59}$$

(60)

$$f_1(t) = \frac{1}{2}S^{-1}\left\{1 + \frac{v}{2} + \frac{v^2}{4} + \frac{v^3}{8} + \frac{v^4}{16} + \cdots + 1\right\} \tag{61}$$

$$f_1(t) = \frac{1}{2}S^{-1}\left\{2 + \frac{v}{2} + \frac{v^2}{4} + \frac{v^3}{8} + \frac{v^4}{16} + \cdots\right\} \tag{62}$$

$$f_1(t) = t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} \tag{63}$$

Therefore, the approximate solution of equation (47) is expressed as

$$f(t) = \sum_{n=0}^{N} f_n(t) = f_0(t) + f_1(t) \tag{64}$$

$$= 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + \cdots \tag{65}$$

Then, the solution can be written in exact form as

$$f(t) = e^t \tag{66}$$

Equation (66) which is the resulting exact solution of equation (47) which is the same as the result obtained in (Mohyud-Din & Yildirim, 2010)

4. CONCLUSION

The Sawi transform technique has been applied to solve delay differential equations (DDEs), producing low error, precise, and effective approximations. In addition to offering a mathematical tool for nonlinear DDEs, the STIM approach is useful for increasing efficiency and accuracy. The application of the Sawi Transform as a dependable technique for getting approximate analytical solutions to nonlinear DDEs is examined in this study. The aim is to address intricate issues that frequently resist straightforward analytical resolutions. The findings of the study demonstrate how well and efficiently the Sawi Transform can approximate answers. Scholars and practitioners dealing with complicated dynamic systems with temporal delays might find the Sawi methodology to be a beneficial option because it has demonstrated its reliability via validation activities and comparative evaluations with different approaches.

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