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THE CRITICAL LOAD OF FUNCTIONALLY GRADED POROUS PLATE WITH LINEAR VARIABLE THICKNESS LAY ON PASTERNAK FOUNDATION

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Abstract: The results for buckling analysis of functionally graded porous plate with linear variable thickness (FGPLVT) laid on Pasternak foundation are presented in this article. The sinusoidal porous distribution is also applied in this structure. With springer stiffness (k_1) and shear stiffness (k_2) as functions of the deflection and its Laplacian, the Pasternak foundation is a two-parameter model that describes the foundation reaction. These numerical results show the important role of the buckling analysis of the FGPLVT plate. Formulations were based on the classical plate theory, with the physical neutral surface considered as the reference plane. Various configurations of clamped, simply supported, and free edges were considered. By implementing the concept of Galerkin's weighted residual method, the fourth-order partial differential governing equation and boundary conditions were converted into two sets of ordinary differential equations, which were then solved numerically using the "Chebfun" numerical computation package.
Keywords: FGPLVT plate, buckling, critical load

1. INTRODUCTION

Due to their ability to improve material strength, high tolerance to temperature shocks, and high strength-to-weight ratio, functionally graded structures have been used in many contemporary engineering applications in recent years. In [1], free vibration analysis of a tapered functionally graded material plate with the inclusion of porosity had been performed. The plate was considered resting on a two-parameter (Winkler and Pasternak) elastic foundation. The displacement model of the kinematic equation for the plates in the paper's formulation was based on the first-order shear deformation theory.

The governing equation for free vibration analysis was obtained using Hamilton's principle. Simple power law, exponential law, and sigmoid law were used to tailor the material properties in the thickness direction of plates. The solution of the resulting partial differential equation was obtained by using Galerkin-Vlasov's method with different boundary conditions. The multi-term extended Kantorovich method was used to solve the bending problem of thin skew functionally graded plate resting on the Winkler elastic foundation under uniformly distributed transverse load as in [2].

Formulations were based on the classical plate theory, with the physical neutral surface considered as the reference plane. Various configurations of clamped, simply supported, and free edges were considered. By implementing the concept of Galerkin's weighted residual method, the fourth-order partial differential governing equation and boundary conditions were converted into two sets of ordinary differential equations, which were then solved numerically using the "Chebfun" numerical computation package. An improved porosity distribution was introduced in [3] for the bending of a novel model of functionally graded sandwich plates via a refined quasi-3D shear and normal deformation theory.

The plates were lying on Pasternak's elastic foundation and exposed to sinusoidal mechanical loads. The shear and normal strains were both included; for that, the shear correction factor was unnecessary. Depending on a specific function, the material properties of the sandwich plates varied continuously across the thickness direction. The equilibrium equations could be derived using the virtual work principle and solved using Navier's method. In [4], the rule of mixture was modified to take into account the effect of porosity and to approximate the material properties assumed to be graded in the thickness direction of the examined annular plate. A semi-analytical model based on Hamilton's principle and spectral analysis was adopted using a homogenization procedure to reduce the problem under consideration to that of an equivalent isotropic homogeneous annular plate. Therefore, it could be very useful to examine the vibration behavior of thin functionally graded annular plates clamped at both edges, including porosities.

The main task of [5] was to further expand the ES-MITC3 for analyzing the buckling characteristics of functionally graded porous variable-thickness plates with sinusoidal porous distribution. The ES-MITC3

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was developed to improve the accuracy of classical triangular elements and overcome the locking phenomenon while still ensuring flexibility in discretizing the structural domain. The first-order shear deformation theory in combination with ES-MITC3 was used due to its simplicity and effectiveness, etc. The paper [6] presented the free vibration behavior of carbon nanotube-reinforced functionally graded composite plates in a thermal environment based on Reddy's higher-order shear deformation theory. The element-free kp-Ritz method was used in the study. In [7], the smoothed four-node element with in-plane rotations MISQ24 was combined with a C0-type higher-order shear deformation theory to propose an improved linear quadrilateral plate element for static and free vibration analyses of laminated composite plates. This improvement resulted in two additional degrees of freedom at each node and required no shear correction factors while ensuring the high precision of numerical solutions. The prime aim of the study [8] was to present analytical formulations and solutions for the buckling analysis of simply supported functionally graded plates using higher-order shear deformation theory without enforcing zero transverse shear stresses on the top and bottom surfaces of the plate. It did not require shear correction factors, and transverse shear stresses varied parabolically across the thickness. The material properties of the plate were assumed to vary in the thickness direction according to a power law distribution in terms of the volume fractions of the constituents. The equations of motion and boundary conditions were derived using the principle of virtual work and so on.

Back to this study, the main goal is to give the critical loads of the functionally graded porous plates with linear variable thickness (FGPLVT) lay on Pasternak foundation by using a simple finite element code in Matlab, respectively. It is given in four sections. Section 1 shows the introduction as above. Section 2 presents the formulations, as well as section 3 shows some essential results. Finally, a few comments are also given in the last section.

2. FGPLVT PLATE LAY ON PASTERNAK FOUNDATION

A functionally graded porous plate $a \times b$ with linear variable thickness ($h_0/h_1/h_2$) (FGPLVT) is depicted in Figure 1.

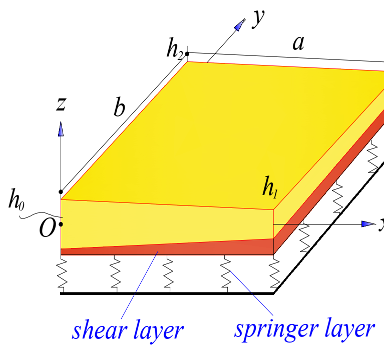

Figure 1. A FGPLVT plate resting on Pasternak foundation

The effective material properties can be expressed as

$$M(z) = \left[(M_c - M_m) V_c(z) + M_m \right] \left[1 - e_0 \cos\left(\frac{\pi z}{h(x,y)} \right) \right] \quad (1)$$

where

$$V_c(z) = \left(\frac{z}{h(x,y)} + \frac{1}{2} \right)^n \quad \text{for} \quad z \in \left[-\frac{h(x,y)}{2}, \frac{h(x,y)}{2} \right] \quad (2)$$

$$h(x) = h_1 + \frac{h_0 - h_1}{a}(a - x) \quad (3)$$

$$h(y) = h_2 + \frac{h_0 - h_2}{b}(b - y) \quad (4)$$

Pasternak's model is determined by

$$\Omega = k_1 w(x,y) - k_2 \left[\left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] \quad (5)$$

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with k_1 is springer stiffness and k_2 is shear stiffness.
Reddy's C^0-TSDT is used to express the plate's displacement field as follows

$$u(x,y,z) = u_0 + \left(z - \frac{4z^3}{3h^2(x,y)}\right)\beta_x - \frac{4z^3}{3h^2(x,y)}\phi_x \quad (6)$$

$$v(x,y,z) = v_0 + \left(z - \frac{4z^3}{3h^2(x,y)}\right)\beta_y - \frac{4z^3}{3h^2(x,y)}\phi_y \quad (7)$$

$$w(x,y,z) = w_0 \quad (8)$$

Using a four-node quadrilateral element with seven degrees of freedom for each node for finite element analysis similar to the literature [6-8] and finding the critical loads.

3. RESULTS

A few numerical examples are presented to close the content of the article with many expectations for future expansion. The material properties can be seen in Table 1.

Table 1. The material properties

Al_2O_3	$E_c = 380$Gpa	$\nu_c = 0.3$	$\rho_c = 3800$ kg/m^3
Al	$E_m = 70$Gpa	$\nu_m = 0.3$	$\rho_m = 2707$ kg/m^3

Firstly, the a/b ratio gets values 0.5, 1, 2 with remaining parameters as $h_0 = a/50$, $h_1 = h_2 = a/75$, $n = 0.5$, and $e_0 = 0.5$. The dimensionless values are presented by $\bar{N}_{cr} = \frac{N_{cr}a^2}{E_m h^3}$, $\bar{k}_1 = \frac{k_1 a^4}{D}$, $\bar{k}_2 = \frac{k_2 a^2}{D}$ with $D = \frac{E_m h^3}{12(1-\nu_m^2)}$. The critical loads of the (CCCC) FGPLVT rectangular plates are shown in Table 2 and compared with other solutions from [5]. It can be seen that the foundation increases the FGPLVT plate stiffness, thus making the dimensionless critical load $\bar{N}_{cr}$ larger.

Observing that the critical load of the FGP plate depends on both the power-law index n and the porosity e_0. Specifically, the ceramic-rich plate will be harder, leading to a larger critical load. Table 3 further lists the critical loads of FGPLVT square plates with input parameters: $h_0 = a/25$, $h_1 = h_2 = a/40$, $\bar{k}_1 = 75$ and $\bar{k}_2 = 15$.

Table 2. The comparison of critical load $\bar{N}_{cr}$

a/b	$\bar{k}_1$	$\bar{k}_2$					
		0		4		10	
		[5]	present	[5]	present	[5]	present
0.5	0	2.7548	2.7612	3.2232	3.2188	3.9156	3.9221
	40	3.0022	3.0109	3.4689	3.4715	4.1588	4.1627
	100	3.3608	3.3594	3.8248	3.8322	4.5105	4.5247
1	0	5.6378	5.6444	6.2269	6.2175	7.0808	7.0768
	40	5.8109	5.8087	6.3888	6.3909	7.2259	7.2300
	100	6.0530	6.0604	6.6140	6.6201	7.4269	7.4211
2	0	15.9689	15.9557	16.5454	16.5337	17.3981	17.3825
	40	16.0101	16.0067	16.5855	16.5904	17.4366	17.4467
	100	16.0716	16.0509	16.6452	16.6416	17.4941	17.5008

Table 3. The critical loads $\bar{N}_{cr}$ of FGPLVT square plates

BCs	e_0	n	Critical load
SSSS	0	0	5.7117
	0.1	1	3.7394
	0.2	2	3.3028
SCSC	0	0	9.9347
	0.3	2	4.6771
	0.5	9	3.8835

4. CONCLUSION

In this study, the critical load of the FGPLVT plate is shown. The results of this article are approximate with other solutions in reference. The results obtained are anticipated to provide valuable insights for the design of FGPLVT plates in practical engineering applications.

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